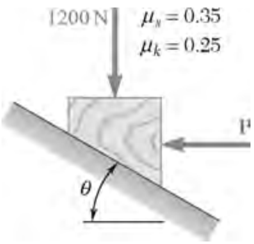
CHAPTER 8

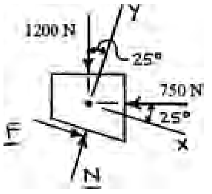


PROBLEM 8.1

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 25°$ and $P = 750$ N.

SOLUTION

Assume equilibrium:



$$\overset{+}{\searrow} \Sigma F_x = 0: \quad F + (1200 \text{ N})\sin 25° - (750 \text{ N})\cos 25° = 0$$

$$F = +172.589 \text{ N} \qquad \mathbf{F} = 172.589 \text{ N} \searrow$$

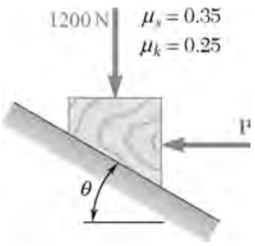
$$+\nearrow \Sigma F_y = 0: \quad N - (1200 \text{ N})\cos 25° - (750 \text{ N})\sin 25° = 0$$

$$N = +1404.53 \text{ N} \qquad \mathbf{N} = 1404.53 \text{ N} \nearrow$$

Maximum friction force: $F_m = \mu_s N = 0.35(1404.53 \text{ N}) = 491.587$ N

Block is in equilibrium ◀

Since $F < F_m$, $\mathbf{F} = 172.6 \text{ N} \searrow$ ◀

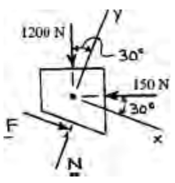


PROBLEM 8.2

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 30°$ and $P = 150$ N.

SOLUTION

<u>Assume equilibrium:</u>



$\searrow\!+\ \Sigma F_x = 0$: $F + (1200\text{ N})\sin 30° - (150\text{ N})\cos 30° = 0$

$F = -470.096$ N $\qquad$ $\mathbf{F} = 470.096$ N $\nwarrow$

$+\nearrow \Sigma F_y = 0$: $N - (1200\text{ N})\cos 30° - (150\text{ N})\sin 30° = 0$

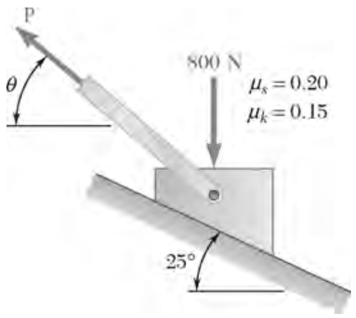
$N = +1114.23$ N $\qquad$ $\mathbf{N} = 1114.23$ N $\nearrow$

Maximum friction force:

$$F_m = \mu_s N = 0.35(1114.23\text{ N}) = 389.981\text{ N}$$

Since **F** is $\nwarrow$ and $F > F_m$, $\qquad$ <u>Block moves down</u> ◀

Actual friction force: $F = F_k = \mu_k N = 0.25(1114.23\text{ N}) = 278.558$ N $\qquad$ $\mathbf{F} = 279$ N $\nwarrow$ ◀

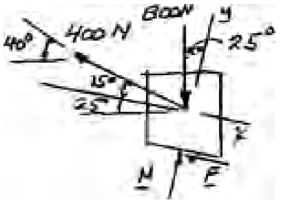


PROBLEM 8.3

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 40°$ and $P = 400$ N.

SOLUTION

<u>Assume equilibrium:</u>



$$+\nearrow \Sigma F_y = 0: \quad N - (800\text{ N})\cos 25° + (400\text{ N})\sin 15° = 0$$

$$N = +621.5\text{ N} \qquad \mathbf{N} = 621.5\text{ N} \uparrow$$

$$+\searrow \Sigma F_x = 0: \quad -F + (800\text{ N})\sin 25° - (400\text{ N})\cos 15° = 0$$

$$F = +48.28\text{ N} \qquad \mathbf{F} = 48.28\text{ N} \nwarrow$$

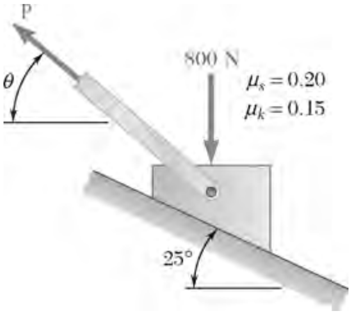
Maximum friction force:

$$\begin{aligned} F_m &= \mu_s N \\ &= 0.20(621.5\text{ N}) \\ &= 124.3\text{ N} \end{aligned}$$

<u>Block is in equilibrium</u> ◀

Since $F < F_m$, $\qquad \mathbf{F} = 48.3\text{ N} \nwarrow$ ◀

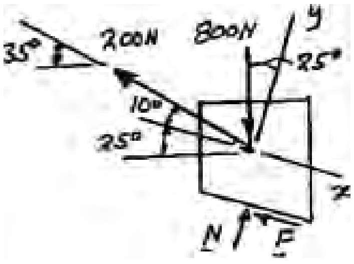
PROBLEM 8.4



Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 35°$ and P = 200 N.

SOLUTION

<u>Assume equilibrium:</u>



$$+\nearrow \Sigma F_y = 0: \quad N - (800 \text{ N})\cos 25° + (200 \text{ N})\sin 10° = 0$$

$$N = 690.3 \text{ N} \qquad \mathbf{N} = 690.3 \text{ N} \uparrow$$

$$\overset{+}{\searrow} \Sigma F_x = 0: \quad -F + (800 \text{ N})\sin 25° - (200 \text{ N})\cos 10° = 0$$

$$F = 141.13 \text{ N} \qquad \mathbf{F} = 141.13 \text{ N} \nwarrow$$

Maximum friction force:

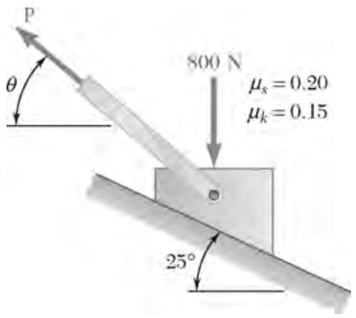
$$\begin{aligned} F_m &= \mu_s N \\ &= (0.20)(690.3 \text{ N}) \\ &= 138.06 \text{ N} \end{aligned}$$

Since $F > F_m$, <u>Block moves down $\searrow$</u> ◀

Friction force:

$$\begin{aligned} F &= \mu_k N \\ &= (0.15)(690.3 \text{ N}) \\ &= 103.547 \text{ N} \end{aligned}$$

$\mathbf{F} = 103.5 \text{ N} \nwarrow$ ◀



PROBLEM 8.5

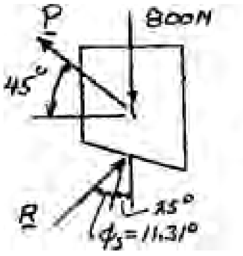
Knowing that $\theta = 45°$, determine the range of values of P for which equilibrium is maintained.

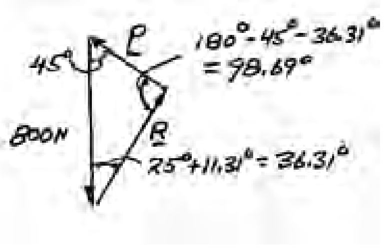
SOLUTION

<u>To start block up the incline:</u>

$$\mu_s = 0.20$$

$$\phi_s = \tan^{-1} 0.20 = 11.31°$$

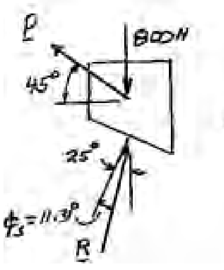


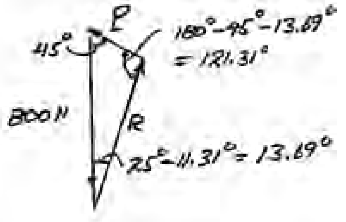


Force triangle:

$$\frac{P}{\sin 36.31°} = \frac{800\text{ N}}{\sin 98.69°} \qquad P = 479.2\text{ N} \triangleleft$$

<u>To prevent block from moving down:</u>

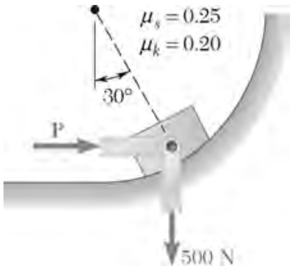




Force triangle:

$$\frac{P}{\sin 13.69°} = \frac{800\text{ N}}{\sin 121.31°} \qquad P = 221.61\text{ N} \triangleleft$$

Equilibrium is maintained for $\qquad 222\text{ N} \le P \le 479\text{ N}$ ◀



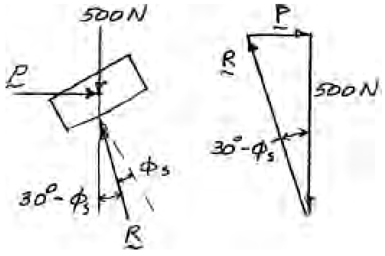
PROBLEM 8.6

Determine the range of values of P for which equilibrium of the block shown is maintained.

SOLUTION

FBD block:

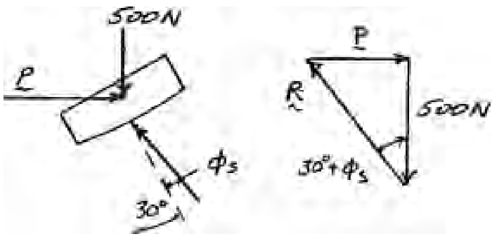
(Impending motion down):



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25$$

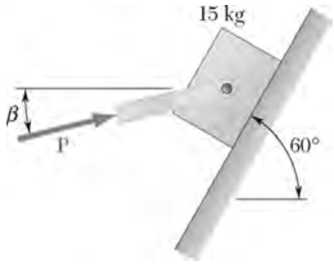
$$P = (500\text{ N})\tan(30° - \tan^{-1}0.25)$$
$$= 143.03\text{ N}$$

(Impending motion up):



$$P = (500\text{ N})\tan(30° + \tan^{-1}0.25)$$
$$= 483.46\text{ N}$$

Equilibrium is maintained for $143.0\text{ N} \leq P \leq 483\text{ N}$ ◀

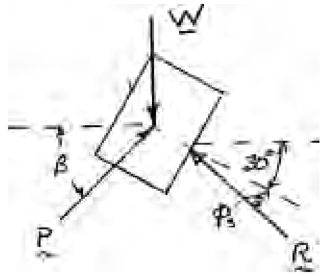


PROBLEM 8.7

Knowing that the coefficient of friction between the 15-kg block and the incline is $\mu_s = 0.25,$ determine (*a*) the smallest value of P required to maintain the block in equilibrium, (*b*) the corresponding value of β.

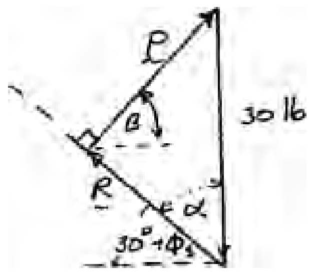
SOLUTION

FBD block (Impending motion downward):



$$W = (15\text{ kg})(9.81\text{ m/s}^2)$$
$$= 147.150\text{ N}$$
$$\phi_s = \tan^{-1}\mu_s$$
$$= \tan^{-1}(0.25)$$
$$= 14.036°$$

(*a*) *Note:* For minimum P, $\quad \mathbf{P} \perp \mathbf{R}$



So

$$\beta = \alpha$$
$$= 90° - (30° + 14.036°)$$
$$= 45.964°$$

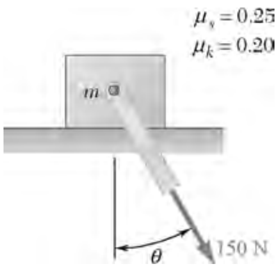
and

$$P = (147.150\text{ N})\sin\alpha$$
$$= (147.150\text{ N})\sin(45.964°)$$

$P = 108.8\text{ N}$ ◀

(*b*) $\beta = 46.0°$ ◀

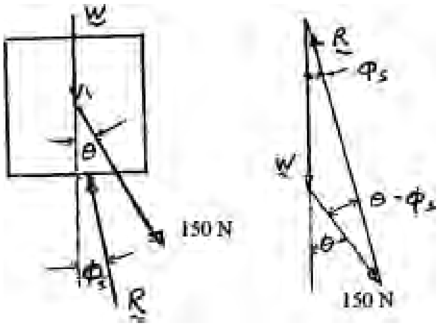
PROBLEM 8.8



Considering only values of θ less than 90°, determine the smallest value of θ required to start the block moving to the right when (*a*) $m = 35$ kg, (*b*) $W = 50$ kg.

SOLUTION

FBD block (Motion impending):



$$\phi_s = \tan^{-1} \mu_s = 14.036°$$

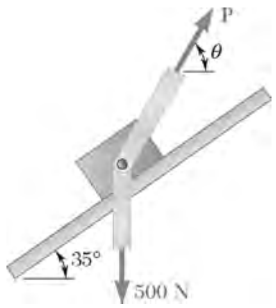
$$\frac{150 \text{ N}}{\sin \phi_s} = \frac{W}{\sin(\theta - \phi_s)}$$

$$\sin(\theta - \phi_s) = \frac{W \sin 14.036°}{150 \text{ N}}$$

or $$\sin(\theta - 14.036°) = \frac{W}{618.476 \text{ N}}$$

(*a*) $m = 35$ kg, $W = 35 \times 9.81$ N: $\theta = 14.036° + \sin^{-1} \dfrac{35 \times 9.81 \text{ N}}{618.476 \text{ N}}$ $\theta = 47.8°$ ◀

(*b*) $m = 50$ kg, $W = 50 \times 9.81$ N: $\theta = 14.036° + \sin^{-1} \dfrac{50 \times 9.81 \text{ N}}{618.476}$ $\theta = 66.5°$ ◀



PROBLEM 8.9

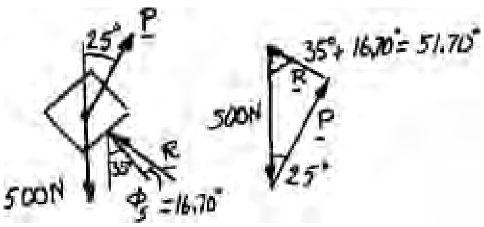
The coefficients of friction between the block and the rail are $\mu_s = 0.30$ and $\mu_k = 0.25$. Knowing that $\theta = 65°$, determine the smallest value of P required (*a*) to start the block moving up the rail, (*b*) to keep it from moving down.

SOLUTION

(*a*) <u>To start block up the rail:</u>

$$\mu_s = 0.30$$

$$\phi_s = \tan^{-1} 0.30 = 16.70°$$

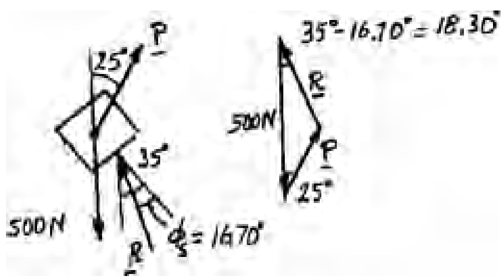


Force triangle:

$$\frac{P}{\sin 51.70°} = \frac{500\text{ N}}{\sin(180° - 25° - 51.70°)}$$

$P = 403$ N ◀

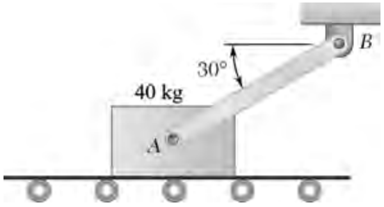
(*b*) <u>To prevent block from moving down:</u>



Force triangle:

$$\frac{P}{\sin 18.30°} = \frac{500\text{ N}}{\sin(180° - 25° - 18.30°)}$$

$P = 229$ N ◀



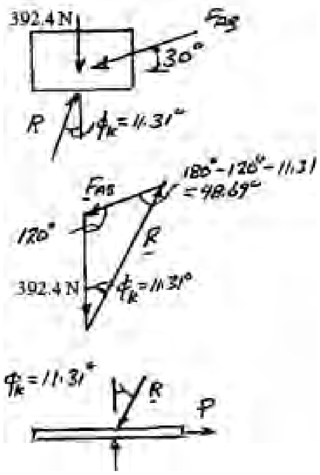
PROBLEM 8.10

The 40 kg block is attached to link *AB* and rests on a moving belt. Knowing that $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the magnitude of the horizontal force **P** that should be applied to the belt to maintain its motion (*a*) to the right, (*b*) to the left.

SOLUTION

We note that link *AB* is a two-force member, since there is motion between belt and block $\mu_k = 0.20$ and $\phi_k = \tan^{-1} 0.20 = 11.31°$. Weight of block = 40 × 9.81 N = 392.4 N

(*a*) Belt moves to right



Free body: Block

Force triangle:

$$\frac{R}{\sin 120°} = \frac{392.4\text{ N}}{\sin 48.69°}$$

$$R = 452.411\text{ N}$$

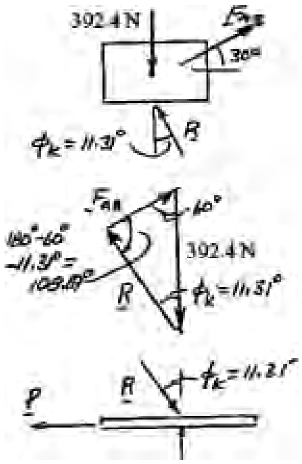
Free body: Belt

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - (452.411)\sin 11.31°$$

$$P = 88.726\text{ N}$$

P = 88.7 N → ◀

(*b*) Belt moves to left



Free body: Block

Force triangle:

$$\frac{R}{\sin 60°} = \frac{392.4}{\sin 108.69°}$$

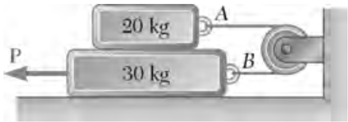
$$R = 358.746\text{ N}$$

Free body: Belt

$$\xrightarrow{+} \Sigma F_x = 0: \quad (358.746\text{ N})\sin 11.31° - P = 0$$

$$P = 70.356\text{ N}$$

P = 70.4 N ← ◀

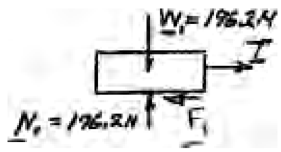


PROBLEM 8.11

The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the smallest force **P** required to start the 30-kg block moving if cable AB (*a*) is attached as shown, (*b*) is removed.

SOLUTION

(*a*) <u>Free body: 20-kg block</u>

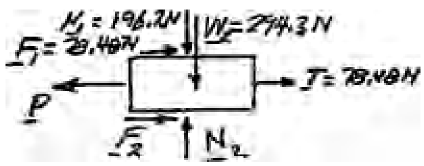


$$W_1 = (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2 \text{ N}$$

$$F_1 = \mu_s N_1 = 0.4(196.2 \text{ N}) = 78.48 \text{ N}$$

$$\overset{+}{\longleftarrow} \Sigma F = 0: \quad T - F_1 = 0 \quad T = F_1 = 78.48 \text{ N}$$

<u>Free body: 30-kg block</u>



$$W_2 = (30 \text{ kg})(9.81 \text{ m/s}^2) = 294.3 \text{ N}$$

$$N_2 = 196.2 \text{ N} + 294.3 \text{ N} = 490.5 \text{ N}$$

$$F_2 = \mu_s N_2 = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

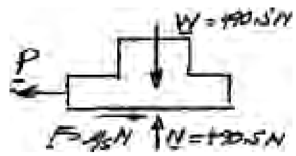
$$\overset{+}{\longleftarrow} \Sigma F = 0: \quad P - F_1 - F_2 - T = 0$$

$$P = 78.48 \text{ N} + 196.2 \text{ N} + 78.48 \text{ N} = 353.2 \text{ N}$$

$\mathbf{P} = 353 \text{ N} \leftarrow$ ◀

(*b*) <u>Free body: Both blocks</u>

<u>Blocks move together</u>

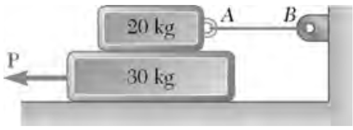


$$W = (50 \text{ kg})(9.81 \text{ m/s}^2) = 490.5 \text{ N}$$

$$\overset{+}{\longleftarrow} \Sigma F = 0: \quad P - F = 0$$

$$P = \mu_s N = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

$\mathbf{P} = 196.2 \text{ N} \leftarrow$ ◀

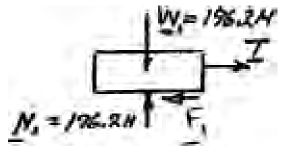


PROBLEM 8.12

The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the smallest force **P** required to start the 30-kg block moving if cable AB (*a*) is attached as shown, (*b*) is removed.

SOLUTION

(*a*) Free body: 20-kg block

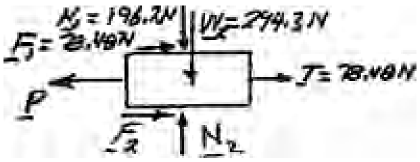


$$W_1 = (20\text{ kg})(9.81\text{ m/s}^2) = 196.2\text{ N}$$

$$F_1 = \mu_s N_1 = 0.4(196.2\text{ N}) = 78.48\text{ N}$$

$$\xrightarrow{+} \Sigma F = 0: \quad T - F_1 = 0 \quad T = F_1 = 78.48\text{ N}$$

Free body: 30-kg block



$$W_2 = (30\text{ kg})(9.81\text{ m/s}^2) = 294.3\text{ N}$$

$$N_2 = 196.2\text{ N} + 294.3\text{ N} = 490.5\text{ N}$$

$$F_2 = \mu_s N_2 = 0.4(490.5\text{ N}) = 196.2\text{ N}$$

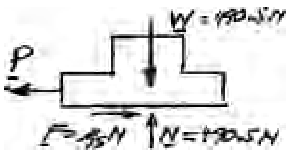
$$\xleftarrow{+} \Sigma F = 0: \quad P - F_1 - F_2 = 0$$

$$P = 78.48\text{ N} + 196.2\text{ N} = 274.7\text{ N}$$

$\mathbf{P} = 275\text{ N} \leftarrow$ ◀

(*b*) Free body: Both blocks

Blocks move together



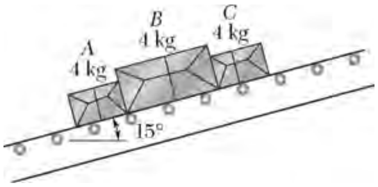
$$W = (50\text{ kg})(9.81\text{ m/s}^2) = 490.5\text{ N}$$

$$\xleftarrow{+} \Sigma F = 0: \quad P - F = 0$$

$$P = \mu_s N = 0.4(490.5\text{ N}) = 196.2\text{ N}$$

$\mathbf{P} = 196.2\text{ N} \leftarrow$ ◀

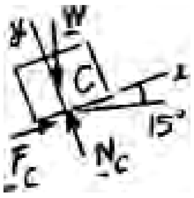
PROBLEM 8.13



Three 4-kg packages *A*, *B*, and *C* are placed on a conveyor belt that is at rest. Between the belt and both packages *A* and *C* the coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.20$; between package *B* and the belt the coefficients are $\mu_s = 0.10$ and $\mu_k = 0.08$. The packages are placed on the belt so that they are in contact with each other and at rest. Determine which, if any, of the packages will move and the friction force acting on each package.

SOLUTION

Consider *C* by itself: Assume equilibrium



$$+\nwarrow \Sigma F_y = 0: \quad N_C - W\cos 15° = 0$$

$$N_C = W\cos 15° = 0.966W$$

$$+\nearrow \Sigma F_x = 0: \quad F_C - W\sin 15° = 0$$

$$F_C = W\sin 15° = 0.259W$$

But $$F_m = \mu_s N_C = 0.30(0.966W) = 0.290W$$

Thus, $F_C < F_m$ <u>Package *C* does not move</u> ◀

$$F_C = 0.259W = 0.259(4 \text{ kg})(9.81 \text{ m/s}^2) = 10.16 \text{ N}$$

$\mathbf{F}_C = 10.16 \text{ N} \nearrow$ ◀

Consider *B* by itself: Assume equilibrium. We find,

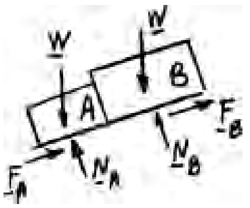
$$F_B = 0.259W$$
$$N_B = 0.966W$$

But $$F_m = \mu_s N_B = 0.10(0.966W) = 0.0966W$$

Thus, $F_B > F_m$. <u>Package *B* would move if alone</u> ◀

PROBLEM 8.13 (Continued)

Consider A and B together: Assume equilibrium



$$F_A = F_B = 0.259W$$
$$N_A = N_B = 0.966W$$
$$F_A + F_B = 2(0.259W) = 0.518W$$
$$(F_A)_m + (F_B)_m = 0.3N_A + 0.1N_B = 0.386W$$

Thus, $F_A + F_B > (F_A)_m + (F_B)_m$ $\quad$ A and B move ◀

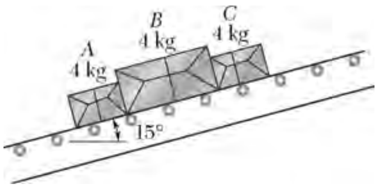
$$F_A = \mu_k N_A = 0.2(0.966)(4)(9.81)$$

$\mathbf{F}_A = 7.58$ N ↗ ◀

$$F_B = \mu_k N_B = 0.08(0.966)(4)(9.81)$$

$\mathbf{F}_B = 3.03$ N ↗ ◀

PROBLEM 8.14

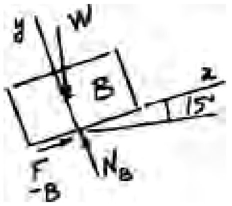


Solve Problem 8.13 assuming that package *B* is placed to the right of both packages *A* and *C*.

PROBLEM 8.13 Three 4-kg packages *A*, *B*, and *C* are placed on a conveyor belt that is at rest. Between the belt and both packages *A* and *C* the coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.20$; between package *B* and the belt the coefficients are $\mu_s = 0.10$ and $\mu_k = 0.08$. The packages are placed on the belt so that they are in contact with each other and at rest. Determine which, if any, of the packages will move and the friction force acting on each package.

SOLUTION

<u>Consider package *B* by itself</u>: Assume equilibrium



$$+\nwarrow \Sigma F_y = 0: \quad N_B - W\cos 15° = 0$$

$$N_B = W\cos 15° = 0.966W$$

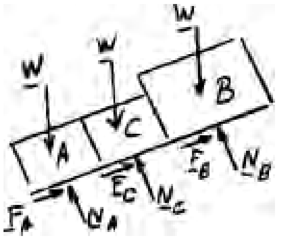
$$+\nearrow \Sigma F_x = 0: \quad F_B - W\sin 15° = 0$$

$$F_B = W\sin 15° = 0.259W$$

But
$$F_m = \mu_s N_B$$
$$= 0.10(0.966W)$$
$$= 0.0966W$$

Thus, $F_B > F_m$. Package *B* would move if alone.

<u>Consider all packages together</u>: Assume equilibrium. In a manner similar to above, we find



$$N_A = N_B = N_C = 0.966W$$
$$F_A = F_B = F_C = 0.259W$$
$$F_A + F_B + F_C = 3(0.259W)$$
$$= 0.777W$$

But
$$(F_A)_m = (F_C)_m = \mu_s N$$
$$= 0.30(0.966W)$$
$$= 0.290W$$

and
$$(F_B)_m = 0.10(0.966W)$$
$$= 0.0966W$$

PROBLEM 8.14 (Continued)

Thus, $(F_A)_m + (F_C)_m + (F_B)_m = 2(0.290W) + 0.0966W$

$= 0.677W$

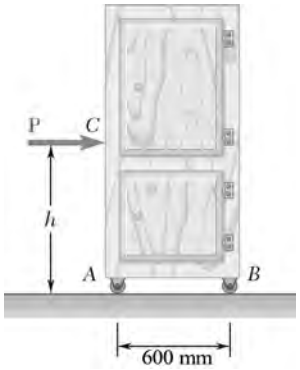
and we note that $F_A + F_B + F_C > (F_A)_m + (F_C)_m + (F_B)_m$

<u>All packages move</u> ◀

$$F_A = F_C = \mu_k N$$
$$= 0.20(0.966)(4 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 7.58 \text{ N}$$
$$F_B = \mu_k N$$
$$= 0.08(0.966)(4 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 3.03 \text{ N}$$

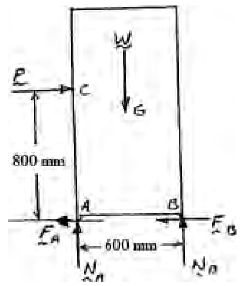
$\mathbf{F}_A = \mathbf{F}_C = 7.58 \text{ N} \nearrow; \ \mathbf{F}_B = 3.03 \text{ N} \nearrow$ ◀

PROBLEM 8.15



A 50 kg cabinet is mounted on casters that can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. If $h = 800$ mm, determine the magnitude of the force **P** required to move the cabinet to the right (*a*) if all casters are locked, (*b*) if the casters at *B* are locked and the casters at *A* are free to rotate, (*c*) if the casters at *A* are locked and the casters at *B* are free to rotate.

SOLUTION



$$W = 50 \times 9.81 \text{ N} = 490.5 \text{ N}$$

FBD cabinet: *Note:* for tipping,

$$N_A = F_A = 0$$

$$\circlearrowleft \Sigma M_B = 0: \quad (300 \text{ mm })W - (800 \text{ mm})P_{\text{tip}} = 0$$

$$P_{\text{tip}} = \frac{3}{8}\text{W} = 183.9375 \text{ N}$$

(*a*) All casters locked. Impending slip:

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0$$

$$N_A + N_B = W \qquad W = 120 \text{ lb}$$

So $\quad F_A + F_B = \mu_s W \qquad \mu_s = 0.3$

$$\rightarrow \Sigma F_x = 0: \quad P - F_A - F_B = 0$$

$$P = F_A + F_B = \mu_s W \qquad P = 0.3(490.5 \text{ N}) \quad \text{or} \quad \mathbf{P} = 147.15 \text{ N} \rightarrow \blacktriangleleft$$

$$(P = 0.3W < P_{\text{tip}} \quad \text{OK})$$

(*b*) Casters at *A* free, so $\quad F_A = 0$

Impending slip: $\quad F_B = \mu_s N_B$

$$\rightarrow \Sigma F_x = 0: \quad P - F_B = 0$$

$$P = F_B = \mu_s N_B \qquad N_B = \frac{P}{\mu_s}$$

PROBLEM 8.15 (Continued)

$$\circlearrowleft \Sigma M_A = 0: \quad (800 \text{ mm})P + (300 \text{ mm})W - (600 \text{ mm})N_B = 0$$

$$8P + 3W - 6\frac{P}{0.3} = 0 \quad P = 0.25W$$

$$(P = 0.25W < P_{\text{tip}} \quad \text{OK})$$

$$P = 0.25(490.5 \text{ N}) = 122.625 \text{ N} \qquad \text{or } \mathbf{P} = 122.6 \text{ N} \rightarrow \blacktriangleleft$$

(*c*) Casters at *B* free, so $F_B = 0$

Impending slip: $F_A = \mu_s N_A$

$$\rightarrow \Sigma F_x = 0: \quad P - F_A = 0 \quad P = F_A = \mu_s N_A$$

$$N_A = \frac{P}{\mu_s} = \frac{P}{0.3}$$

$$\circlearrowright \Sigma M_B = 0: \quad (300 \text{ mm})W - (800 \text{ mm})P - (600 \text{ mm})N_A = 0$$

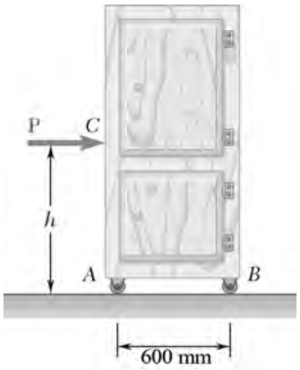
$$3W - 8P - 6\frac{P}{0.3} = 0$$

$$P = 0.107143W = 52.5536 \text{ N}$$

$$(P < P_{\text{tip}} \quad \text{OK})$$

$$\mathbf{P} = 52.6 \text{ N} \rightarrow \blacktriangleleft$$

PROBLEM 8.16

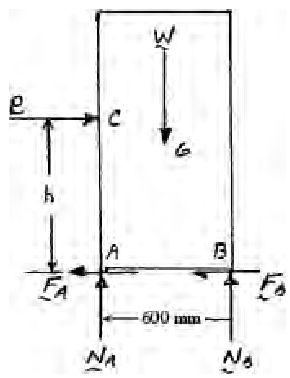


A 50-kg cabinet is mounted on casters that can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. Assuming that the casters at both A and B are locked, determine (*a*) the force **P** required to move the cabinet to the right, (*b*) the largest allowable value of h if the cabinet is not to tip over.

SOLUTION

$$W = (50 \times 9.81)\text{N} = 490.5\text{ N}$$

FBD cabinet:



(*a*)

$$\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0$$

$$N_A + N_B = W$$

Impending slip:

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

So

$$F_A + F_B = \mu_s W$$

$$\rightarrow \Sigma F_x = 0: \quad P - F_A - F_B = 0$$

$W = 120$ lb

$\mu_s = 0.3$

$$P = F_A + F_B = \mu_s W$$

$$P = 0.3(490.5\text{ N}) = 147.15\text{ N}$$

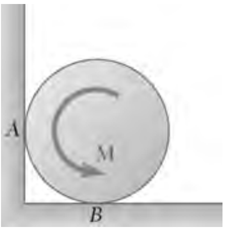
$\mathbf{P} = 147.2\text{ N} \rightarrow$ ◀

(*b*) For tipping, $N_A = F_A = 0$

$$\circlearrowleft \Sigma M_B = 0: \quad hP - (300\text{ mm})W = 0$$

$$h_{\max} = (300\text{ mm})\frac{W}{P} = (300\text{ mm})\frac{1}{\mu_s} = \frac{300\text{ mm}}{0.3}$$

$h_{\max} = 1000\text{ mm}$ ◀

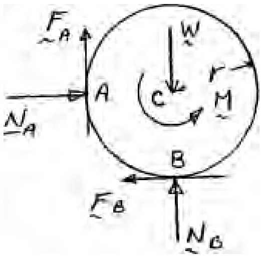


PROBLEM 8.17

The cylinder shown is of weight W and radius r, and the coefficient of static friction μ_s is the same at A and B. Determine the magnitude of the largest couple **M** that can be applied to the cylinder if it is not to rotate.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

$$\rightarrow \Sigma F_x = 0: \quad N_A - F_B = 0$$

$$N_A = F_B = \mu_s N_B$$

$$F_A = \mu_s N_A = \mu_s^2 N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0$$

$$N_B + \mu_s^2 N_B = W$$

or

$$N_B = \frac{W}{1+\mu_s^2}$$

and

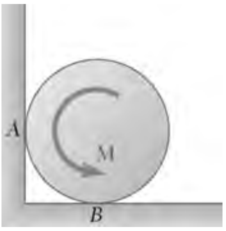
$$F_B = \frac{\mu_s W}{1+\mu_s^2}$$

$$F_A = \frac{\mu_s^2 W}{1+\mu^2}$$

$$\circlearrowright \Sigma M_C = 0: \quad M - r(F_A + F_B) = 0$$

$$M = r\left(\mu_s + \mu_s^2\right)\frac{W}{1+\mu_s^2}$$

$$M_{\max} = Wr\mu_s \frac{1+\mu_s}{1+\mu_s^2} \blacktriangleleft$$

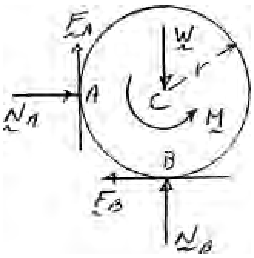


PROBLEM 8.18

The cylinder shown is of weight W and radius r. Express in terms W and r the magnitude of the largest couple **M** that can be applied to the cylinder if it is not to rotate, assuming the coefficient of static friction to be (*a*) zero at A and 0.30 at B, (*b*) 0.25 at A and 0.30 at B.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_A N_A$$
$$F_B = \mu_B N_B$$

$$\rightarrow \Sigma F_x = 0: \quad N_A - F_B = 0$$
$$N_A = F_B = \mu_B N_B$$
$$F_A = \mu_A N_A = \mu_A \mu_B N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0$$
$$N_B(1 + \mu_A \mu_B) = W$$

or
$$N_B = \frac{1}{1 + \mu_A \mu_B} W$$

and
$$F_B = \mu_B N_B = \frac{\mu_B}{1 + \mu_A \mu_B} W$$

$$F_A = \mu_A \mu_B N_B = \frac{\mu_A \mu_B}{1 + \mu_A \mu_B} W$$

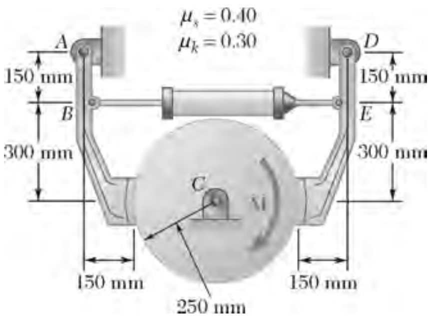
$$\circlearrowright \Sigma M_C = 0: \quad M - r(F_A + F_B) = 0$$
$$M = Wr\mu_B \frac{1 + \mu_A}{1 + \mu_A \mu_B}$$

(*a*) For $\mu_A = 0$ and $\mu_B = 0.30$:

$$M = 0.300Wr \blacktriangleleft$$

(*b*) For $\mu_A = 0.25$ and $\mu_B = 0.30$:

$$M = 0.349Wr \blacktriangleleft$$



PROBLEM 8.19

The hydraulic cylinder shown exerts a force of 3 kN directed to the right on Point *B* and to the left on Point *E*. Determine the magnitude of the couple **M** required to rotate the drum clockwise at a constant speed.

SOLUTION

Free body: Drum

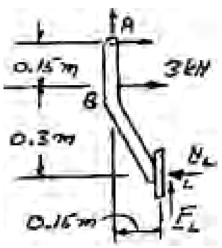
$$+\circlearrowright \Sigma M_C = 0: \quad M - (0.25\text{ m})(F_L + F_R) = 0$$

$$M = (0.25\text{ m})(F_L + F_R) \qquad (1)$$

Since drum is rotating

$$F_L = \mu_k N_L = 0.3 N_L$$
$$F_R = \mu_k N_R = 0.3 N_R$$

Free body: Left arm *ABL*



$$+\circlearrowleft \Sigma M_A = 0: \quad (3\text{ kN})(0.15\text{ m}) + F_L(0.15\text{ m}) - N_L(0.45\text{ m}) = 0$$

$$0.45\text{ kN}\cdot\text{m} + (0.3N_L)(0.15\text{ m}) - N_L(0.45\text{ m}) = 0$$

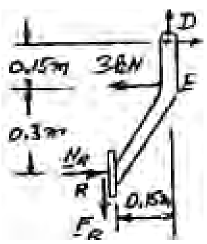
$$0.405 N_L = 0.45$$

$$N_L = 1.111\text{ kN}$$

$$F_L = 0.3N_L = 0.3(1.111\text{ kN})$$

$$= 0.3333\text{ kN} \qquad (2)$$

Free body: Right arm *DER*



$$+\circlearrowright \Sigma M_D = 0: \quad (3\text{ kN})(0.15\text{ m}) - F_R(0.15\text{ m}) - N_R(0.45\text{ m}) = 0$$

$$0.45\text{ kN}\cdot\text{m} - (0.3N_R)(0.15\text{ m}) - N_R(0.45\text{ m}) = 0$$

$$0.495 N_R = 0.45$$

$$N_R = 0.9091\text{ kN}$$

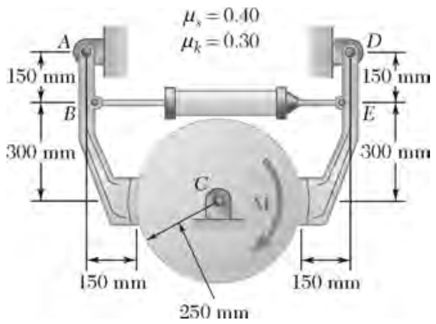
$$F_R = \mu_k N_R = 0.3(0.9091\text{ kN})$$

$$= 0.2727\text{ kN} \qquad (3)$$

Substitute for F_L and F_R into (1): $\quad M = (0.25\text{ m})(0.333\text{ kN} + 0.2727\text{ kN})$

$$M = 0.1515\text{ kN}\cdot\text{m}$$

$$M = 151.5\text{ N}\cdot\text{m} \blacktriangleleft$$



PROBLEM 8.20

A couple **M** of magnitude 100 N m is applied to the drum as shown. Determine the smallest force that must be exerted by the hydraulic cylinder on joints *B* and *E* if the drum is not to rotate.

SOLUTION

Free body: Drum

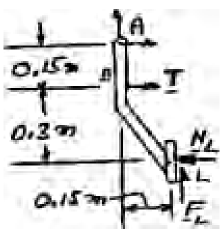
$$+\circlearrowleft \Sigma M_C = 0: \quad 100 \text{ N} \cdot \text{m} - (0.25 \text{ m})(F_L + F_R) = 0$$

$$F_L + F_R = 400 \text{ N} \qquad (1)$$

Since motion impends

$$F_L = \mu_s N_L = 0.4 N_L$$
$$F_R = \mu_s N_R = 0.4 N_R$$

Free body: Left arm *ABL*



$$+\circlearrowleft \Sigma M_A = 0: \quad T(0.15 \text{ m}) + F_L(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

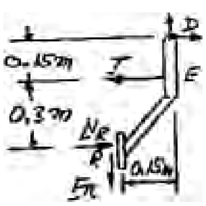
$$0.15T + (0.4N_L)(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

$$0.39N_L = 0.15T; \quad N_L = 0.38462T$$

$$F_L = 0.4N_L = 0.4(0.38462T)$$

$$F_L = 0.15385T \qquad (2)$$

Free body: Right arm *DER*



$$+\circlearrowleft \Sigma M_D = 0: \quad T(0.15 \text{ m}) - F_R(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

$$0.15T - (0.4N_R)(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

$$0.51N_R = 0.15T; \quad N_R = 0.29412T$$

$$F_R = 0.4N_R = 0.4(0.29412T)$$

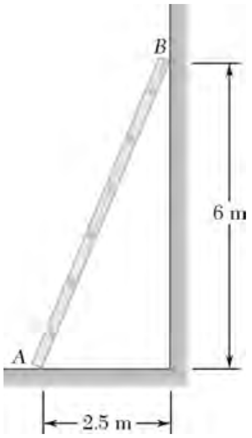
$$F_R = 0.11765T \qquad (3)$$

Substitute for F_L and F_R into Eq. (1):

$$0.15385T + 0.11765T = 400$$

$$T = 1473.3 \text{ N}$$

$$T = 1.473 \text{ kN} \blacktriangleleft$$



PROBLEM 8.21

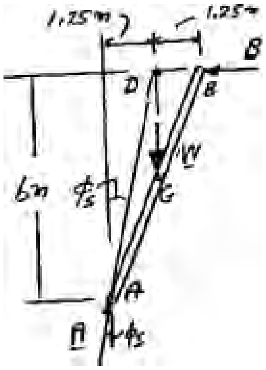
A 6.5-m ladder *AB* leans against a wall as shown. Assuming that the coefficient of static friction μ_s is zero at *B*, determine the smallest value of μ_s at *A* for which equilibrium is maintained.

SOLUTION

Free body: Ladder

Three-force body.

Line of action of **A** must pass through *D*, where **W** and **B** intersect.

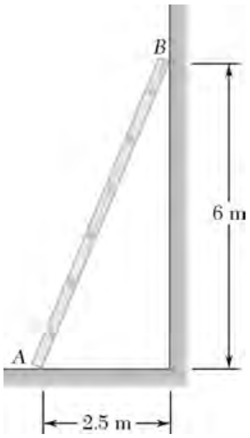


At *A*:

$$\mu_s = \tan\phi_s = \frac{1.25 \text{ m}}{6 \text{ m}} = 0.2083$$

$$\mu_s = 0.208 \blacktriangleleft$$

PROBLEM 8.22

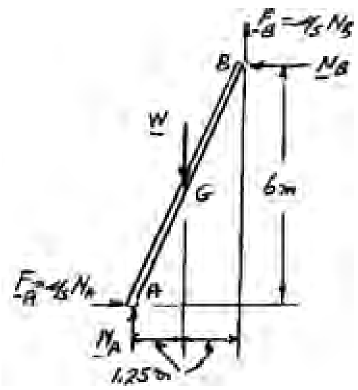


A 6.5-m ladder AB leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at A and B, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

Free body: Ladder

Motion impending:



$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

$$+\circlearrowleft \Sigma M_A = 0: \quad W(1.25 \text{ m}) - N_B(6 \text{ m}) - \mu_s N_B(2.5 \text{ m}) = 0$$

$$N_B = \frac{1.25W}{6 + 2.5\mu_s} \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad N_A + \mu_s N_B - W = 0$$

$$N_A = W - \mu_s N_B$$

$$N_A = W - \frac{1.25\mu_s W}{6 + 2.5\mu_s} \tag{2}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad \mu_s N_A - N_B = 0$$

Substitute for N_A and N_B from Eqs. (1) and (2):

$$\mu_s W - \frac{1.25\mu_s^2 W}{6 + 2.5\mu_s} = \frac{1.25W}{6 + 2.5\mu_s}$$

$$6\mu_s + 2.5\mu_s^2 - 1.25\mu_s^2 = 1.25$$

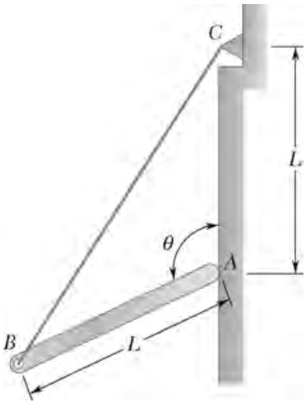
$$1.25\mu_s^2 + 6\mu_s - 1.25 = 0$$

$$\mu_s = 0.2$$

and $$\mu_s = -5 \quad \text{(Discard)}$$

$$\mu_s = 0.200 \blacktriangleleft$$

PROBLEM 8.23

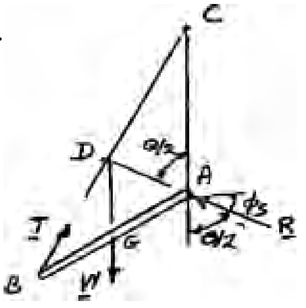


End A of a slender, uniform rod of length L and weight W bears on a surface as shown, while end B is supported by a cord BC. Knowing that the coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine (*a*) the largest value of θ for which motion is impending, (*b*) the corresponding value of the tension in the cord.

SOLUTION

Free-body diagram

Three-force body. Line of action of **R** must pass through D, where **T** and **R** intersect.



Motion impends:

$$\tan\phi_s = 0.4$$

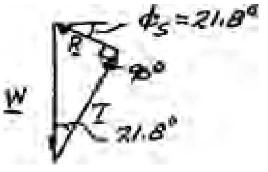
$$\phi_s = 21.80°$$

(*a*) Since $BG = GA$, it follows that $BD = DC$ and AD bisects $\angle BAC$

$$\frac{\theta}{2} + \phi_s = 90°$$

$$\frac{\theta}{2} + 21.8° = 90°$$

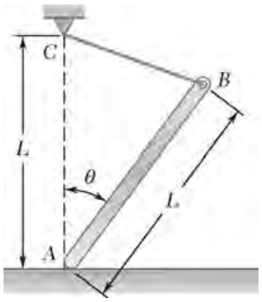
$\theta = 136.4°$ ◀



(*b*) Force triangle (right triangle):

$$T = W \cos 21.8°$$

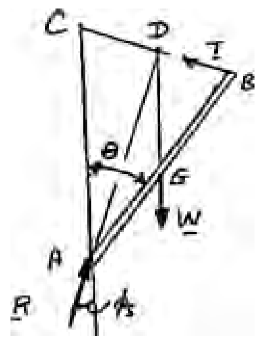
$T = 0.928W$ ◀



PROBLEM 8.24

End A of a slender, uniform rod of length L and weight W bears on a surface as shown, while end B is supported by a cord BC. Knowing that the coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine (a) the largest value of θ for which motion is impending, (b) the corresponding value of the tension in the cord.

SOLUTION



Free-body diagram

Rod AB is a three-force body. Thus, line of action of **R** must pass through D, where **W** and **T** intersect.

Since $AG = GB$, $CD = DB$ and the median AD of the isosceles triangle ABC bisects the angle θ.

(a) Thus,

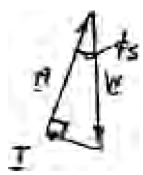
$$\phi_s = \frac{1}{2}\theta$$

Since motion impends,

$$\phi_s = \tan^{-1} 0.40 = 21.80^\circ$$
$$\theta = 2\phi_s = 2(21.8^\circ)$$

$\theta = 43.6^\circ$ ◀

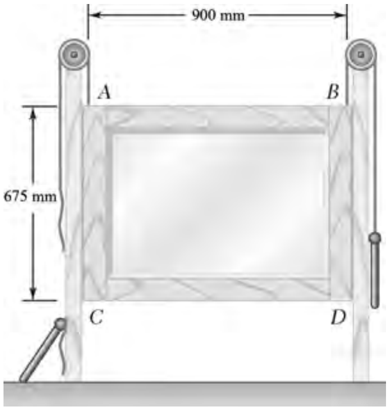
(b) Force triangle:



This is a right triangle.

$$T = W \sin\phi_s$$
$$= W \sin 21.8^\circ$$

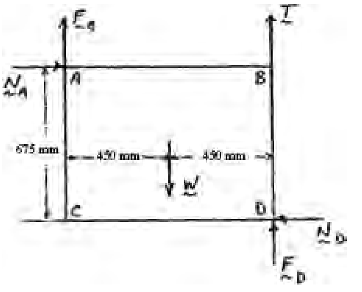
$T = 0.371W$ ◀



PROBLEM 8.25

A window sash of mass 5 kg is normally supported by two 2.5 kg sash weights. Knowing that the window remains open after one sash cord has broken, determine the smallest possible value of the coefficient of static friction. (Assume that the sash is slightly smaller than the frame and will bind only at Points *A* and *D*.)

SOLUTION



FBD window: $T = 2.5 \times 9.81 = 24.525 \text{ N} = \dfrac{W}{2}$ (for given data)

$$\rightarrow \Sigma F_x = 0: \quad N_A - N_D = 0$$

$$N_A = N_D$$

Impending motion:

$$F_A = \mu_s N_A$$

$$F_D = \mu_s N_D$$

$$\circlearrowleft \Sigma M_D = 0: \quad (450 \text{ mm})W - (675 \text{ mm})N_A - (900 \text{ mm})F_A = 0 \quad W = 5 \times 9.81 = 49.05 \text{ N}$$

$$W = \frac{3}{2}N_A + 2\mu_s N_A$$

$$N_A = \frac{2W}{3 + 4\mu_s}$$

$$\uparrow \Sigma F_y = 0: \quad F_A - W + T + F_D = 0$$

$$F_A + F_D = W - T = \frac{W}{2}$$

Now

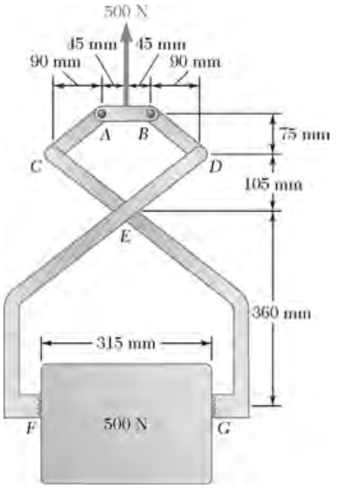
$$F_A + F_D = \mu_s(N_A + N_D) = 2\mu_s N_A$$

Then

$$\frac{W}{2} = 2\mu_s \frac{2W}{3 + 4\mu_s}$$

or

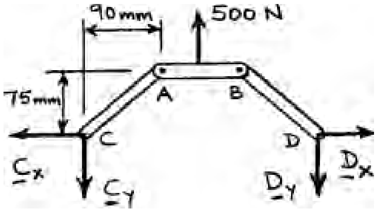
$$\mu_s = 0.750 \blacktriangleleft$$



PROBLEM 8.26

A 500-N concrete block is to be lifted by the pair of tongs shown. Determine the smallest allowable value of the coefficient of static friction between the block and the tongs at *F* and *G*.

SOLUTION



Free body: Members *CA*, *AB*, *BD*

By symmetry: $C_y = D_y = \frac{1}{2}(500) = 250\text{ N}$

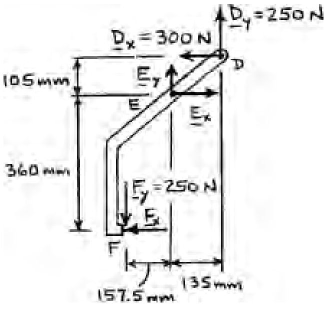
Since *CA* is a two-force member,

$$\frac{C_x}{90\text{ mm}} = \frac{C_y}{75\text{ mm}} = \frac{250\text{ N}}{75\text{ mm}}$$

$$C_x = 300\text{ N}$$

$$\Sigma F_x = 0:\quad D_x = C_x$$

$$D_x = 300\text{ N}$$



Free body: Tong *DEF*

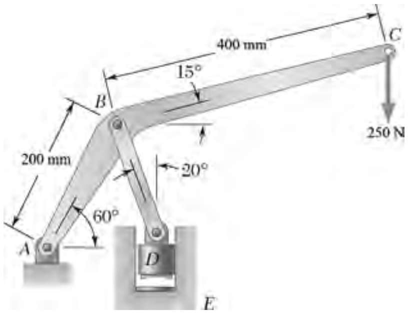
$$+\circlearrowleft \Sigma M_E = 0:\quad (300\text{ N})(105\text{ mm}) + (250\text{ N})(135\text{ mm})$$

$$+ (250\text{ N})(157.5\text{ mm}) - F_x(360\text{ mm}) = 0$$

$$F_x = +290.625\text{ N}$$

Minimum value of μ_s: $\mu_s = \frac{F_y}{F_x} = \frac{250\text{ N}}{290.625\text{ N}}$

$\mu_s = 0.860$ ◀

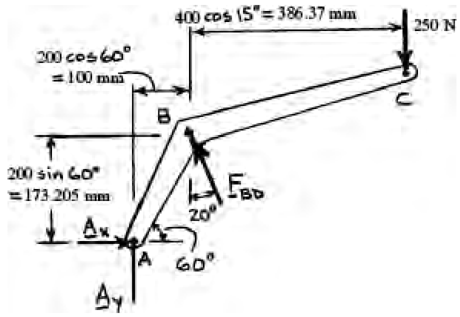


PROBLEM 8.27

The press shown is used to emboss a small seal at E. Knowing that the coefficient of static friction between the vertical guide and the embossing die D is 0.30, determine the force exerted by the die on the seal.

SOLUTION

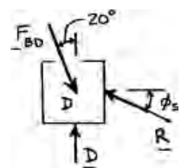
<u>Free body: Member *ABC*</u>



$$\circlearrowleft\!\!+\ \Sigma M_A = 0: \quad F_{BD}\cos 20°(100\,\text{mm}) + F_{BD}\sin 20°(173.205\,\text{mm})$$

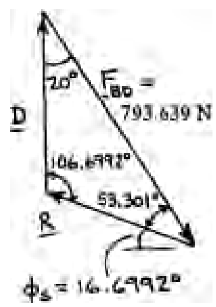
$$-(250\text{ N})(100 + 386.37\,\text{mm}) = 0$$

$$F_{BD} = 793.639\text{ N}$$



<u>Free body: Die *D*</u>

$$\phi_s = \tan^{-1}\mu_s$$
$$= \tan^{-1} 0.3$$
$$= 16.6992°$$

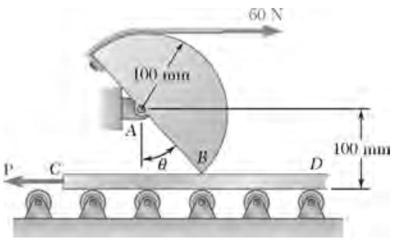


Force triangle:

$$\frac{D}{\sin 53.301°} = \frac{793.639\text{ N}}{\sin 106.6992°}$$

$$D = 664.347\text{ N}$$

On seal: $\mathbf{D} = 664\text{ N}\downarrow$ ◀

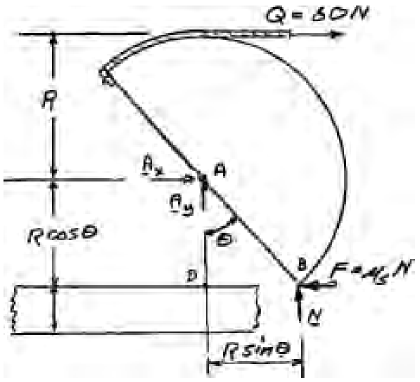


PROBLEM 8.28

The 100-mm-radius cam shown is used to control the motion of the plate *CD*. Knowing that the coefficient of static friction between the cam and the plate is 0.45 and neglecting friction at the roller supports, determine (*a*) the force **P** required to maintain the motion of the plate, knowing that the plate is 20 mm thick, (*b*) the largest thickness of the plate for which the mechanism is self locking (i.e., for which the plate cannot be moved however large the force **P** may be).

SOLUTION

Free body: Cam



Impending motion: $F = \mu_s N$

$$+\circlearrowleft \Sigma M_A = 0: \quad QR - NR\sin\theta + (\mu_s N)R\cos\theta = 0$$

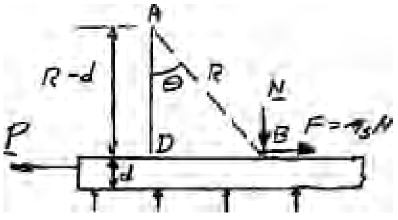
$$N = \frac{Q}{\sin\theta - \mu_s\cos\theta} \qquad (1)$$

Free body: Plate $\quad \Sigma F_x = 0 \quad P = \mu_s N \qquad (2)$

Geometry in ABD with $R = 100$ mm and $d = 20$ mm

$$\cos\theta = \frac{R-d}{R} = \frac{80\text{ mm}}{100\text{ mm}} = 0.8$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = 0.6$$



PROBLEM 8.28 (Continued)

(*a*) Eq. (1) using $Q = 60 \text{ N} \quad \text{and} \quad \mu_s = 0.45$

$$N = \frac{60 \text{ N}}{0.6 - (0.45)(0.8)}$$

$$= \frac{60}{0.24} = 250 \text{ N}$$

Eq. (2) $P = \mu_s N = (0.45)(250 \text{ N})$

$$P = 112.5 \text{ N} \blacktriangleleft$$

(*b*) <u>For *P* = </u>, $N = \infty$. Denominator is zero in Eq. (1).

$$\sin\theta - \mu_s \cos\theta = 0$$

$$\tan\theta = \mu_s = 0.45$$

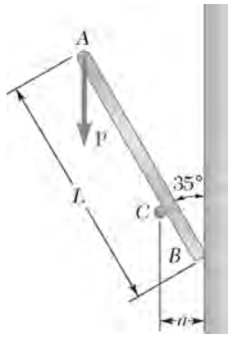
$$\theta = 24.23°$$

$$\cos\theta = \frac{R - d}{R}$$

$$\cos 24.23 = \frac{100 - d}{100}$$

$$d = 8.81 \text{ mm} \blacktriangleleft$$

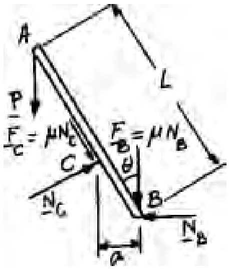
PROBLEM 8.29



A slender rod of length L is lodged between peg C and the vertical wall and supports a load **P** at end A. Knowing that the coefficient of static friction is 0.20 at both B and C, find the range of values of the ratio L/a for which equilibrium is maintained.

SOLUTION

We shall first assume that the motion of end B is *impending upward*. The friction forces at B and C will have the values and directions indicated in the *FB* diagram.



$$+\circlearrowleft \Sigma M_B = 0: \quad PL\sin\theta - N_C\left(\frac{a}{\sin\theta}\right) = 0$$

$$N_C = \frac{PL}{a}\sin^2\theta \tag{1}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_C\cos\theta + \mu N_C\sin\theta - N_B = 0 \tag{2}$$

$$+\uparrow \Sigma F_y = 0: \quad N_C\sin\theta - \mu N_C\cos\theta - \mu N_B - P = 0 \tag{3}$$

Multiply Eq. (2) by μ and subtract from Eq. (3):

$$N_C(\sin\theta - \mu\cos\theta) - \mu N_C(\cos\theta + \mu\sin\theta) - P = 0$$

$$P = N_C[\sin\theta(1-\mu^2) - 2\mu\cos\theta]$$

Substitute for N_C from Eq. (1) and solve for a/L:

$$\frac{a}{L} = \sin^2\theta[(1-\mu^2)\sin\theta - 2\mu\cos\theta] \tag{4}$$

Making $\theta = 35°$ and $\mu = 0.20$ in Eq. (4):

$$\frac{a}{L} = \sin^2 35°[(1-0.04)\sin 35° - 2(0.20)\cos 35°]$$
$$= 0.07336$$

$$\frac{L}{a} = 13.63 \quad \triangleleft$$

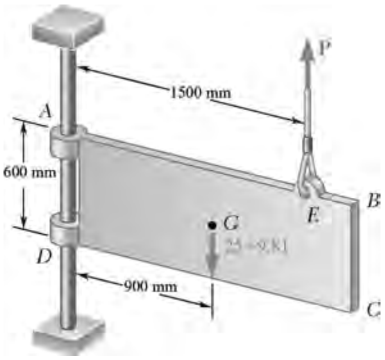
PROBLEM 8.29 (Continued)

Assuming now that the motion at B is *impending downward*, we should reverse the direction of $\mathbf{F}_B$ and $\mathbf{F}_C$ in the *FB* diagram. The same result may be obtained by making $\theta = 35°$ and $\mu = -0.20$ in Eq. (4):

$$\frac{a}{L} = \sin^2 35°[(1-0.04)\sin 35° - 2(-0.20)\cos 35°]$$
$$= 0.2889$$

$$\frac{L}{a} = 3.461 \quad \triangleleft$$

Thus, the range of values of L/a for which equilibrium is maintained is $3.46 \le \dfrac{L}{a} \le 13.63$ ◀

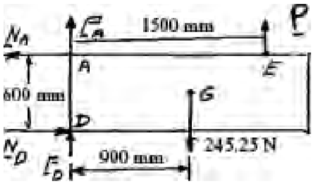


PROBLEM 8.30

The 25 kg plate *ABCD* is attached at *A* and *D* to collars that can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at *E* is (*a*) $P = 0$, (*b*) $P = 100$ N.

SOLUTION

(*a*) <u>P = 0</u> W = (25 × 9.81)N = 245.25 N



$$+\circlearrowleft \Sigma M_D = 0: \quad N_A(600 \text{ mm}) - (245.25 \text{ N})(900 \text{ mm}) = 0$$

$$N_A = 367.875 \text{ N}$$

$$\Sigma F_x = 0: \quad N_D = N_A = 367.875 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_D - 245.25 \text{ N} = 0$$

$$F_A + F_D = 245.25 \text{ N}$$

But: $(F_A)_m = \mu_s N_A = 0.40(367.875 \text{ N}) = 147.15 \text{ N}$

$(F_D)_m = \mu_s N_D = 0.40(367.875 \text{ N}) = 147.15 \text{ N}$

Thus: $(F_A)_m + (F_D)_m = 294.3 \text{ N}$

and $(F_A)_m + (F_D)_m > F_A + F_D$ Plate is in equilibrium ◀

(*b*) <u>P = 100 N</u> $+\circlearrowleft \Sigma M_D = 0: \quad N_A(600 \text{ mm}) - (245.25 \text{ N})(900 \text{ mm}) + (100 \text{ N})(1500 \text{ mm}) = 0$

$$N_A = 117.875 \text{ N}$$

$$\Sigma F_x = 0: \quad N_D = N_A = 117.875 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_D - 245.25 \text{ N} + 100 \text{ N} = 0$$

$$F_A + F_D = 145.25 \text{ N}$$

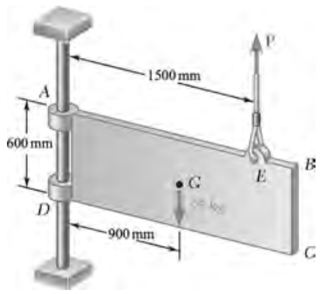
But: $(F_A)_m = \mu_s N_A = 0.4(117.875 \text{ N}) = 47.15 \text{ N}$

$(F_D)_m = \mu_s N_D = 0.4(117.875 \text{ N}) = 47.15 \text{ N}$

Thus: $(F_A)_m + (F_D)_m = 94.3 \text{ N}$

and $F_A + F_D > (F_A)_m + (F_D)_m$ Plate moves downward ◀

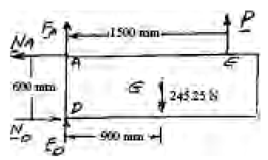
PROBLEM 8.31



In Problem 8.30, determine the range of values of the magnitude P of the vertical force applied at E for which the plate will move downward.

PROBLEM 8.30 The 25-kg plate *ABCD* is attached at *A* and *D* to collars that can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at *E* is (*a*) $P = 0$, (*b*) $P = 100\text{ N}$.

SOLUTION



We shall consider the following two cases:

(1) $0 < P < 147.15\text{ N}$

$$+\circlearrowleft \Sigma M_D = 0: \quad N_A(600\text{ mm}) - (245.25\text{ N})(900\text{ mm}) + P(1500\text{ mm}) = 0$$

$$N_A = 367.875\text{ N} - 2.5P$$

(*Note:* $N_A \geq 0$ and directed for $P \leq 147.15\text{ N}$ as assumed here)

$$\Sigma F_x = 0: \quad N_A = N_D$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_D + P - 245.25 = 0$$

$$F_A + F_D = 245.25 - P$$

But:

$$(F_A)_m = (F_D)_m = \mu_s N_A$$
$$= 0.40(367.875 - 2.5P)$$
$$= (147.15 - P)$$

Plate moves if: $F_A + F_D > (F_A)_m + (F_D)_m$

or $245.25 - P > (147.15 - P) + (147.15 - P)$ $\qquad P > 49.05\text{ N} \triangleleft$

(2) $147.15\text{ N} < P < 245.25\text{ N}$

$$+\circlearrowleft \Sigma M_D = 0: \quad -N_A(600\text{ mm}) - (245.25\text{ N})(900\text{ mm}) + P(1500\text{ mm}) = 0$$

$$N_A = 2.5P - 367.875$$

(*Note:* $N_A >$ and directed for $P > 147.15\text{ N}$ as assumed)

$$\Sigma F_x = 0: \quad N_A = N_D$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_D + P - 245.25 = 0$$

$$F_A + F_D = 245.25 - P$$

PROBLEM 8.31 (Continued)

But: $(F_A)_m = (F_D)_m = \mu_s N_A$

$= 0.40(2.5P - 367.875)$

$= P - 147.15 \text{ N}$

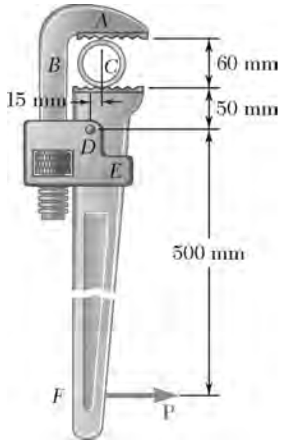
Plate moves if: $F_A + F_D > (F_A)_m + (F_D)_m$

$$245.25 - P > (P - 147.15) + (P - 147.15) \qquad P < 179.85 \text{ N} \triangleleft$$

Thus, plate moves downward for: $49.1 \text{ N} < P < 179.9 \text{ N}$ ◀

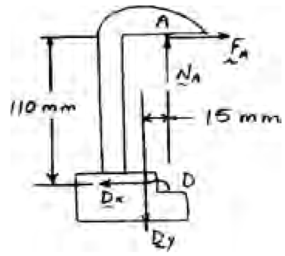
(*Note:* For $P > 245.25 \text{ N}$, plate is in equilibrium)

PROBLEM 8.32



A pipe of diameter 60 mm is gripped by the stillson wrench shown. Portions *AB* and *DE* of the wrench are rigidly attached to each other, and portion *CF* is connected by a pin at *D*. If the wrench is to grip the pipe and be self-locking, determine the required minimum coefficients of friction at *A* and *C*.

SOLUTION



FBD *ABD*:

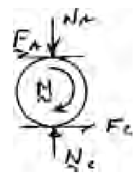
$$\circlearrowright \Sigma M_D = 0: \quad (15 \text{ mm})N_A - (110 \text{ mm})F_A = 0$$

Impending motion: $F_A = \mu_A N_A$

Then $15 - 110\mu_A = 0$

or $\mu_A = 0.13636$ $\qquad \mu_A = 0.1364$ ◀

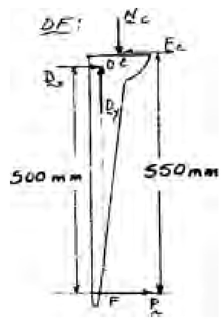
$$\Sigma F_x = 0: \quad F_A - D_x = 0 \quad D_x = F_A$$



FBD pipe:

$$\uparrow \Sigma F_y = 0: \quad N_C - N_A = 0$$

$$N_C = N_A$$



FBD *DF*:

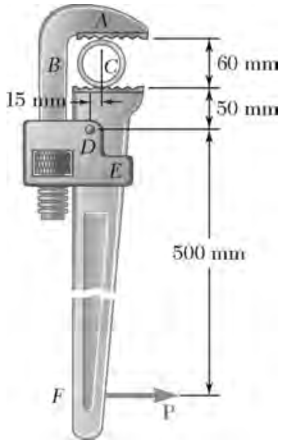
$$\circlearrowright \Sigma M_F = 0: \quad (550 \text{ mm})F_C - (15 \text{ mm})N_C - (500 \text{ mm})D_x = 0$$

Impending motion: $F_C = \mu_C N_C$

Then $550\mu_C - 15 = 500\dfrac{F_A}{N_C}$

But $N_C = N_A$ and $\dfrac{F_A}{N_A} = \mu_A = 0.13636$

So $550\mu_C = 15 + 500(0.13636)$ $\qquad \mu_C = 0.1512$ ◀



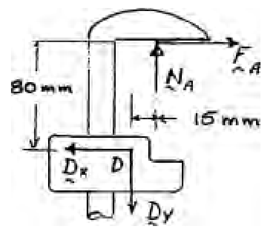
PROBLEM 8.33

Solve Problem 8.32 assuming that the diameter of the pipe is 30 mm.

PROBLEM 8.32 A pipe of diameter 60 mm is gripped by the stillson wrench shown. Portions *AB* and *DE* of the wrench are rigidly attached to each other, and portion *CF* is connected by a pin at *D*. If the wrench is to grip the pipe and be self-locking, determine the required minimum coefficients of friction at *A* and *C*.

SOLUTION

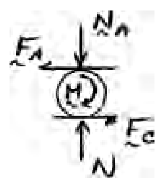
FBD *ABD*:



$$\circlearrowleft \Sigma M_D = 0: \quad (15 \text{ mm})N_A - (80 \text{ mm})F_A = 0$$

Impending motion: $F_A = \mu_A N_A$

Then $15 \text{ mm} - (80 \text{ mm})\mu_A = 0$ $\qquad \mu_A = 0.1875$ ◀

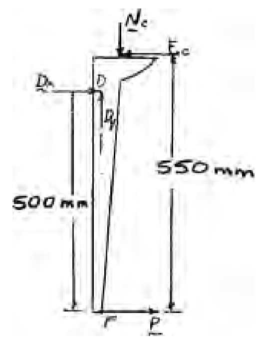


$$\Sigma F_x = 0: \quad F_A - D_x = 0$$

So that $D_x = F_A = 0.1875 N_A$

FBD pipe: $\uparrow \Sigma F_y = 0: \quad N_C - N_A = 0$

$$N_C = N_A$$



FBD *DF*: $\circlearrowleft \Sigma M_F = 0: \quad (550 \text{ mm})F_C - (15 \text{ mm})N_C - (500 \text{ mm})D_x = 0$

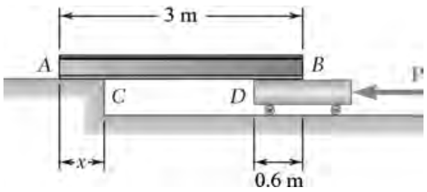
Impending motion: $F_C = \mu_C N_C$

$$550\mu_C - 15 = 500(0.1875)\frac{N_A}{N_C}$$

But $N_A = N_C$ (from pipe *FBD*) so $\dfrac{N_A}{N_C} = 1$

and $\qquad \mu_C = 0.1977$ ◀

PROBLEM 8.34

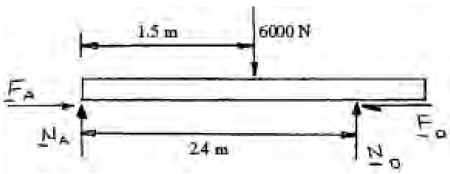


A 3 m beam, weighing 6000 N, is to be moved to the left onto the platform. A horizontal force **P** is applied to the dolly, which is mounted on frictionless wheels. The coefficients of friction between all surfaces are $\mu_s = 0.30$ and $\mu_k = 0.25$, and initially $x = 0.6$ m. Knowing that the top surface of the dolly is slightly higher than the platform, determine the force **P** required to start moving the beam. (*Hint:* The beam is supported at *A* and *D*.)

SOLUTION

The beam is in contact with dolly at point D.

FBD beam:



$$+\circlearrowleft \Sigma M_A = 0: \quad N_D(2.4 \text{ m}) - (6000 \text{ N})(1.5 \text{ m}) = 0$$

$$\mathbf{N}_D = 3750 \text{ N}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad N_A - 6000 \text{ N} + 3750 \text{ N} = 0$$

$$\mathbf{N}_A = 2250 \text{ N}\uparrow$$

$$(F_A)_m = \mu_s N_A = 0.3(2250) = 675 \text{ N}$$

$$(F_D)_m = \mu_s N_D = 0.3(3750) = 1125 \text{ N}$$

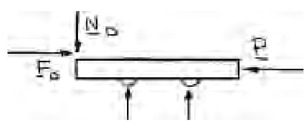
Since $(F_A)_m < (F_D)_m$, sliding first impends at *A* with

$$F_A = (F_A)_m = 675 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_A - F_D = 0$$

$$F_D = F_A = 675 \text{ N}$$

FBD dolly:



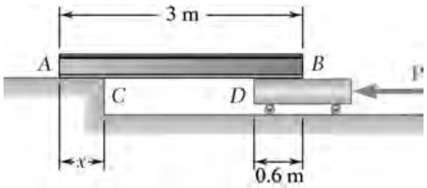
From *FBD* of dolly:

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_D - P = 0$$

$$P = F_D = 675 \text{ N}$$

$P = 675$ N ◀

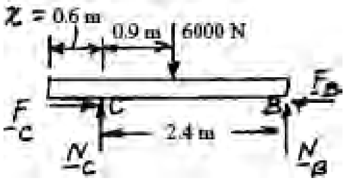
PROBLEM 8.35



(a) Show that the beam of Problem 8.34 *cannot* be moved if the top surface of the dolly is slightly *lower* than the platform. (b) Show that the beam *can* be moved if two 900 N workers stand on the beam at B and determine how far to the left the beam can be moved.

PROBLEM 8.34 A 3 m beam, weighing 6000 N, is to be moved to the left onto the platform. A horizontal force **P** is applied to the dolly, which is mounted on frictionless wheels. The coefficients of friction between all surfaces are $\mu_s = 0.30$ and $\mu_k = 0.25$, and initially $x = 0.6\,\text{m}$. Knowing that the top surface of the dolly is slightly higher than the platform, determine the force **P** required to start moving the beam. (*Hint:* The beam is supported at A and D.)

SOLUTION



The beam is in contact with dolly at point B.

(a) Beam alone

$$+\circlearrowleft \Sigma M_C = 0: \quad N_B(2.4\text{ m}) - (6000\text{ N})(0.9\text{ m}) = 0$$

$$\mathbf{N}_B = 2250\text{ N} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad N_C + 2250 - 6000 = 0$$

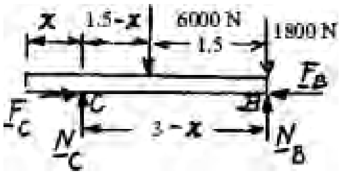
$$\mathbf{N}_C = 3750\text{ N} \uparrow$$

$$(F_C)_m = \mu_s N_C = 0.3(3750) = 1125\text{ N}$$

$$(F_B)_m = \mu_s N_B = 0.3(2250) = 675\text{ N}$$

Since $(F_B)_m < (F_C)_m$, sliding first impends at B, where the dolly will move and the Beam cannot be moved

(b) Beam with workers standing at B



$$+\circlearrowleft \Sigma M_C = 0: \quad N_B(3-x) - (6000)(1.5-x) + 1800(3-x) = 0$$

$$N_B = \frac{14400 - 7800x}{3-x}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (6000)(1.5) - N_C(3-x) = 0$$

$$N_C = \frac{9000}{3-x}$$

PROBLEM 8.35 (Continued)

Check that beam starts moving for $x = 0.6$ m:

For $x = 0.6$ m:

$$N_B = \frac{14400 - 7800(0.6)}{3 - 0.6} = 4050 \text{ N}$$

$$N_C = \frac{9000}{3 - 0.6} = 3750 \text{ N}$$

$$(F_C)_m = \mu_s N_C = 0.3(3750) = 1125 \text{ N}$$

$$(F_B)_m = \mu_s N_B = 0.3(4050) = 1215 \text{ N}$$

Since $(F_C)_m < (F_B)_m$, sliding first impends at C, Beam moves ◀

How far does beam move?

Beam will stop moving when

$$F_C = (F_B)_m$$

But

$$F_C = \mu_k N_C = 0.25\frac{9000}{3 - x} = \frac{2250}{3 - x}$$

and

$$(F_B)_m = \mu_s N_B = 0.30\frac{14400 - 7800x}{3 - x} = \frac{4320 - 2340x}{3 - x}$$

Setting $F_C = (F_B)_m$:

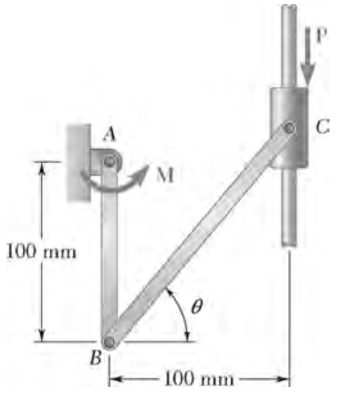
$$2250 = 4320 - 2340x$$

$x = 0.885$ m ◀

$$\Rightarrow x = \frac{23}{26} \text{ m} = 0.88462 \text{ m}$$

(*Note:* We have assumed that, once started, motion is continuous and uniform (no acceleration).)

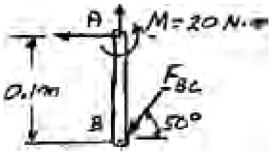
PROBLEM 8.36



Knowing that the coefficient of static friction between the collar and the rod is 0.35, determine the range of values of P for which equilibrium is maintained when $\theta = 50°$ and $M = 20\ \text{N}\cdot\text{m}$.

SOLUTION

Free body member *AB*:

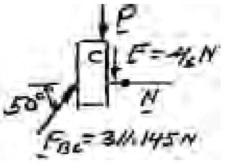


BC is a two-force member.

$$\circlearrowleft^{+}\ \Sigma M_A = 0:\quad 20\ \text{N}\cdot\text{m} - F_{BC}\cos 50°(0.1\ \text{m}) = 0$$

$$F_{BC} = 311.145\ \text{N}$$

<u>Motion of *C* impending upward</u>:



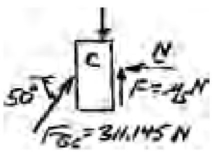
$$\xrightarrow{+}\ \Sigma F_x = 0:\quad (311.145\ \text{N})\cos 50° - N = 0$$

$$N = 200\ \text{N}$$

$$+\!\uparrow \Sigma F_y = 0:\quad (311.145\ \text{N})\sin 50° - P - (0.35)(200\ \text{N}) = 0$$

$$P = 168.351\ \text{N} \ \triangleleft$$

<u>Motion of *C* impending downward</u>:



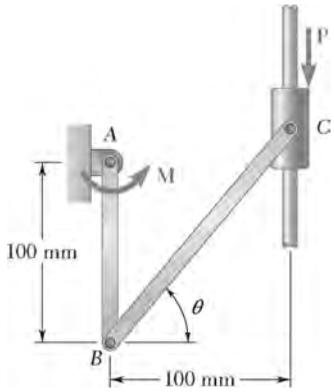
$$\xrightarrow{+}\ \Sigma F_x = 0:\quad (311.145\ \text{N})\cos 50° - N = 0$$

$$N = 200\ \text{N}$$

$$+\!\uparrow \Sigma F_y = 0:\quad (311.145\ \text{N})\sin 50° - P + (0.35)(200\ \text{N}) = 0$$

$$P = 308.35\ \text{N} \ \triangleleft$$

<u>Range of *P*</u>: $168.4\ \text{N} \le P \le 308\ \text{N}$ ◀

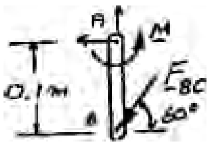


PROBLEM 8.37

Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of M for which equilibrium is maintained when $\theta = 60°$ and $P = 200$ N.

SOLUTION

Free body member *AB*:

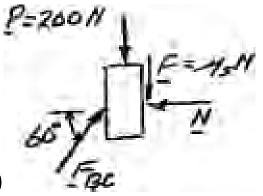


BC is a two-force member.

$$\circlearrowleft^{+} \Sigma M_A = 0: \quad M - F_{BC} \cos 60°(0.1 \text{ m}) = 0$$

$$M = 0.05 F_{BC} \tag{1}$$

<u>Motion of C impending upward</u>:



$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BC} \cos 60° - N = 0$$

$$N = 0.5 F_{BC}$$

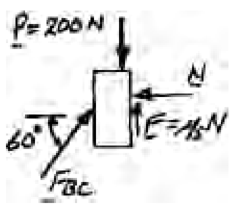
$$+\uparrow \Sigma F_y = 0: \quad F_{BC} \sin 60° - 200 \text{ N} - (0.40)(0.5 F_{BC}) = 0$$

$$F_{BC} = 300.29 \text{ N}$$

Eq. (1): $\quad M = 0.05(300.29)$

$$M = 15.014 \text{ N} \cdot \text{m} \quad \triangleleft$$

<u>Motion of C impending downward</u>:



$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BC} \cos 60° - N = 0$$

$$N = 0.5 F_{BC}$$

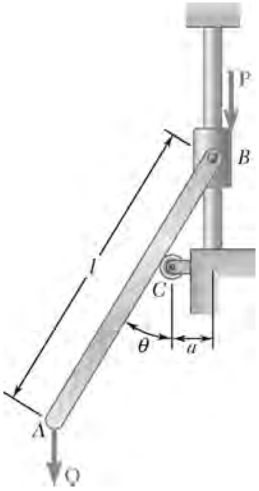
$$+\uparrow \Sigma F_y = 0: \quad F_{BC} \sin 60° - 200 \text{ N} + (0.40)(0.5 F_{BC}) = 0$$

$$F_{BC} = 187.613 \text{ N}$$

Eq. (1): $\quad M = 0.05(187.613)$

$$M = 9.381 \text{ N.m} \quad \triangleleft$$

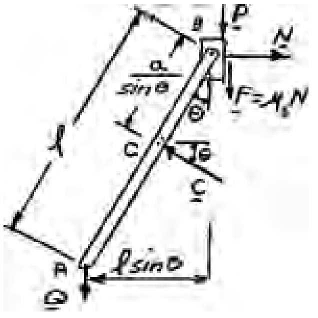
Range of M: $\quad 9.38 \text{ N} \cdot \text{m} \leq M \leq 15.01 \text{ N} \cdot \text{m} \quad \blacktriangleleft$



PROBLEM 8.38

The slender rod AB of length $l = 600$ mm is attached to a collar at B and rests on a small wheel located at a horizontal distance $a = 80$ mm from the vertical rod on which the collar slides. Knowing that the coefficient of static friction between the collar and the vertical rod is 0.25 and neglecting the radius of the wheel, determine the range of values of P for which equilibrium is maintained when $Q = 100$ N and $\theta = 30°$.

SOLUTION



For motion of collar at B impending upward:

$$\mathbf{F} = \mu_s N$$

$$+\circlearrowleft \Sigma M_B = 0: \quad Ql\sin\theta - \frac{Ca}{\sin\theta} = 0$$

$$C = Q\left(\frac{l}{a}\right)\sin^2\theta$$

$$\Sigma F_x = 0: \quad N = C\cos\theta = Q\left(\frac{l}{a}\right)\sin^2\theta\cos\theta$$

$$+\downarrow \Sigma F_y = 0: \quad P + Q - C\sin\theta - \mu_s N = 0$$

$$P + Q - Q\left(\frac{l}{a}\right)\sin^3\theta - \mu_s Q\left(\frac{l}{a}\right)\sin^2\theta\cos\theta = 0$$

$$P = Q\left[\frac{l}{a}\sin^2\theta(\sin\theta - \mu_s\cos\theta) - 1\right] \tag{1}$$

Substitute data:

$$P = (100\text{ N})\left[\frac{600\text{ mm}}{80\text{ mm}}\sin^2 30°(\sin 30° - 0.25\cos 30°) - 1\right]$$

$$P = -46.84\text{ N} \quad (P \text{ is directed } \uparrow)$$

$P = -46.8$ N ◁

PROBLEM 8.38 (Continued)

For motion of collar, impending downward:

$$\mathbf{F} = \mu_s N \uparrow$$

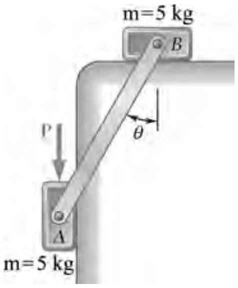
In Eq. (1) we substitute $-\mu_s$ for μ_s.

$$P = Q\left[\frac{l}{a}\sin^2\theta(\sin\theta + \mu_s\cos\theta) - 1\right]$$

$$P = (100\ \text{N})\left[\frac{600\ \text{mm}}{80\ \text{mm}}\sin^2 30°(\sin 30° + 0.25\cos\theta) - 1\right]$$

$$P = +34.34\ \text{N} \quad \triangleleft$$

For equilibrium: $-46.8\ \text{N} \le P \le 34.3\ \text{N}$ ◀



PROBLEM 8.39

Two 5 kg blocks A and B are connected by a slender rod of negligible weight. The coefficient of static friction is 0.30 between all surfaces of contact, and the rod forms an angle $\theta = 30°$ with the vertical. (a) Show that the system is in equilibrium when $P = 0$. (b) Determine the largest value of P for which equilibrium is maintained.

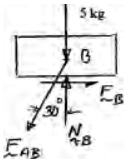
SOLUTION

Weight of each block = 5 g

FBD block B:

(a) Since $P = 13.20$ N to initiate motion (see part b) equilibrium exists with $P = 0$ ◄

(b) For $P_{\max}$, motion impends at both surfaces:



Block B: $\uparrow \Sigma F_y = 0$: $N_B - 5\text{g} - F_{AB}\cos 30° = 0$

$$N_B = 5\text{ g} + \frac{\sqrt{3}}{2} F_{AB} \qquad (1)$$

Impending motion: $F_B = \mu_s N_B = 0.3 N_B$

$\Sigma F_x = 0$: $F_B - F_{AB}\sin 30° = 0$

$$F_{AB} = 2F_B = 0.6N_B \qquad (2)$$

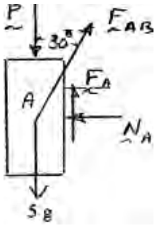
Solving Eqs. (1) and (2): $N_B = 5\text{g} + \frac{\sqrt{3}}{2}(0.6N_B) \Rightarrow \boldsymbol{N_B} = 102.106\text{ N}$

FBD block A:

Then $F_{AB} = 0.6N_B = 61.2636\text{ N}$

Block A: $\Sigma F_x = 0$: $F_{AB}\sin 30° - N_A = 0$

$$N_A = \frac{1}{2}F_{AB} = \frac{1}{2}(61.2636\text{ N}) = 30.6318\text{ N}$$



Impending motion: $F_A = \mu_s N_A = 0.3(30.6318\text{ N}) = 9.18954\text{ N}$

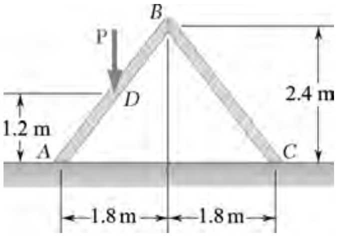
$\uparrow \Sigma F_y = 0$: $F_A + F_{AB}\cos 30° - P - 5\text{g} = 0$

$$P = F_A + \frac{\sqrt{3}}{2}F_{AB} - 5\text{ g}$$

$$= 9.18954\text{ N} + \frac{\sqrt{3}}{2}(61.2636\text{ N}) - 5 \times 9.81\text{ N}$$

$$= 13.1954\text{ N}$$

$P = 13.20$ N ◄



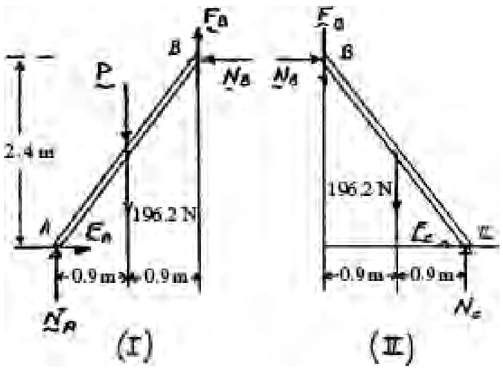
PROBLEM 8.40

Two identical uniform boards, each of weight 20 kg, are temporarily leaned against each other as shown. Knowing that the coefficient of static friction between all surfaces is 0.40, determine (*a*) the largest magnitude of the force **P** for which equilibrium will be maintained, (*b*) the surface at which motion will impend.

SOLUTION

Weight of each plank = 20 × 9.81 = 196.2 N

Board FBDs:



Assume impending motion at C, so

$$F_C = \mu_s N_C = 0.4 N_C$$

FBD II: $\circlearrowright \Sigma M_B = 0{:}\quad (1.8 \text{ m})N_C - (2.4 \text{ m})F_C - (0.9 \text{ m})(196.2 \text{ N}) = 0$

$$[1.8 \text{ m} - 0.4(2.4 \text{ m})]N_C = (0.9 \text{ m})(196.2 \text{ N})$$

or $$N_C = 210.214 \text{ N}$$

and $$F_C = 0.4 N_C = 84.0857 \text{ N}$$

$$\Sigma F_x = 0{:}\quad N_B - F_C = 0$$

$$N_B = F_C = 84.0857 \text{ N}$$

$$\uparrow \Sigma F_y = 0{:}\quad -F_B - 196.2 \text{ N} + N_C = 0$$

$$F_B = N_C - 196.2 \text{ N} = 14.014 \text{ N}$$

Check for motion at B: $$\frac{F_B}{N_B} = \frac{14.014 \text{ N}}{84.0857 \text{ N}} = 0.167 < \mu_s, \text{ OK, no motion.}$$

PROBLEM 8.40 (Continued)

FBD I: $\circlearrowright \Sigma M_A = 0\!: \quad (2.4\text{ m})N_B + (1.8\text{ m})F_B - (0.9\text{ m})(P + 196.2\text{ N}) = 0$

$$P = \frac{(2.4\text{ m})(84.0857\text{ N}) + (1.8\text{ m})(14.014\text{ N})}{0.9\text{ m}} - 196.2\text{ N}$$
$$= 56.0565\text{ N}$$

Check for slip at A (unlikely because of P):

$$\Sigma F_x = 0\!: \quad F_A - N_B = 0 \quad \text{or} \quad F_A = N_B = 84.0857\text{ N}$$

$$\uparrow \Sigma F_y = 0\!: \quad N_A - P - 196.2\text{ N} + F_B = 0$$

or $$N_A = 56.0565\text{ N} + 196.2\text{ N} - 14.014\text{ N}$$
$$= 238.2425\text{ N}$$

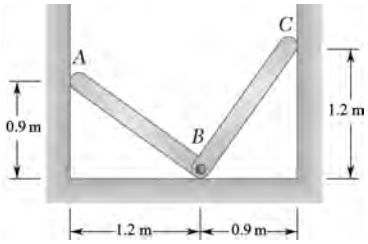
Then $$\frac{F_A}{N_A} = \frac{84.0857\text{ N}}{238.2425\text{ N}} = 0.353 < \mu_s$$

OK, no slip assumption is correct

Therefore

(a) $P_{\max} = 56.1\text{ N}$ ◀

(b) Motion impends at C ◀

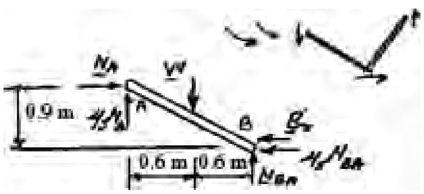


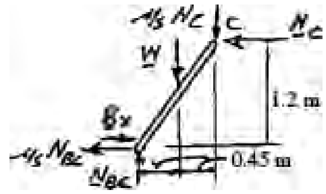
PROBLEM 8.41

Two identical 1.5 m-long rods connected by a pin at B are placed between two walls and a horizontal surface as shown. Denoting by μ_s the coefficient of static friction at A, B, and C, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

Sense of impending motion:





$$\circlearrowleft + \Sigma M_B = 0: \quad 0.6W - 0.9N_A - 1.2\mu_s N_A = 0$$

$$N_A = \frac{2W}{(3+4\mu_s)} \qquad (1)$$

$$\Sigma F_y: \quad N_{AB} = W - \mu_s N_A \qquad (3)$$

$$\xleftarrow{+} \Sigma F_x = 0: \quad B_x + \mu_s N_{BA} - N_A = 0$$

$$B_x = N_A - \mu_s N_{BA} \qquad (5)$$

$$\circlearrowright + \Sigma M_B = 0: \quad 0.45W - 1.2N_C + 0.9\mu_s N_C = 0$$

$$N_C = \frac{1.5W}{(4-3\mu_s)} \qquad (2)$$

$$\Sigma F_y: \quad N_{BC} = W + \mu_s N_C \qquad (4)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x - N_C - \mu_s N_{BC} = 0$$

$$B_x = N_C + \mu_s N_{BC} \qquad (6)$$

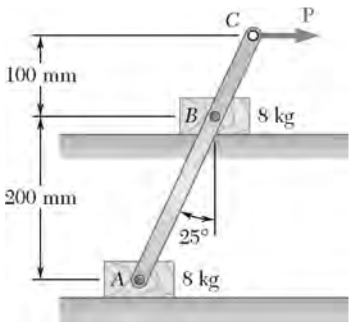
Equate (5) and (6):
$$N_A - \mu_s N_{BA} = N_C + \mu_s N_{BC}$$

Substitute from Eqs. (3) and (4):
$$N_A - \mu_s(W - \mu_s N_A) = N_C + \mu_s(W + \mu_s N_C)$$
$$N_A(1+\mu_s^2) - \mu_s W = N_C(1+\mu_s^2) + \mu_s W$$

Substitute from Eqs. (1) and (2):
$$\frac{2W}{3+4\mu_s}(1+\mu_s^2) - \mu_s W = \frac{1.5W}{4-3\mu_s}(1+\mu_s^2) + \mu_s W$$
$$\frac{2}{3+4\mu_s} - \frac{1.5}{4-3\mu_s} = \frac{2\mu_s}{1+\mu_s^2}$$

Solve for μ_s: $\mu_s = 0.0949$ ◄

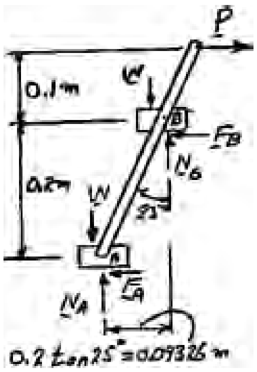
PROBLEM 8.42



Two 8-kg blocks A and B resting on shelves are connected by a rod of negligible mass. Knowing that the magnitude of a horizontal force **P** applied at C is slowly increased from zero, determine the value of P for which motion occurs, and what that motion is, when the coefficient of static friction between all surfaces is (a) $\mu_s = 0.40$, (b) $\mu_s = 0.50$.

SOLUTION

(a) $\underline{\mu_s = 0.40}$: Assume blocks slide to right.



$$W = mg = (8 \text{ kg})(9.81 \text{ m/s}^2) = 78.48 \text{ N}$$

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

$$+\uparrow \Sigma F_y = 0: \quad N_A + N_B - 2W = 0$$

$$N_A + N_B = 2W$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - F_A - F_B = 0$$

$$P = F_A + F_B = \mu_s(N_A + N_B) = \mu_s(2W) \qquad (1)$$

$$P = 0.40(2)(78.48 \text{ N}) = 62.78 \text{ N}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad P(0.1 \text{ m}) - (N_A - W)(0.09326 \text{ m}) + F_A(0.2 \text{ m}) = 0$$

$$(62.78)(0.1) - (N_A - 78.48)(0.09326) + (0.4)(N_B)(0.2) = 0$$

$$0.17326 N_A = 1.041$$

$$N_A = 6.01 \text{ N} > 0 \quad \text{OK}$$

<u>System slides</u>: $P = 62.8$ N ◀

(b) $\underline{\mu_s = 0.50}$: See part a.

Eq. (1): $\quad P = 0.5(2)(78.48 \text{ N}) = 78.48 \text{ N}$

$$+\circlearrowleft \Sigma M_B = 0: \quad P(0.1 \text{ m}) + (N_A - W)(0.09326 \text{ m}) + F_A(0.2 \text{ m}) = 0$$

$$(78.48)(0.1) + (N_A - 78.48)(0.09326) + (0.5)N_A(0.2) = 0$$

$$0.19326 N_A = -0.529$$

$$N_A = -2.73 \text{ N} < 0 \text{ uplift, rotation about } B$$

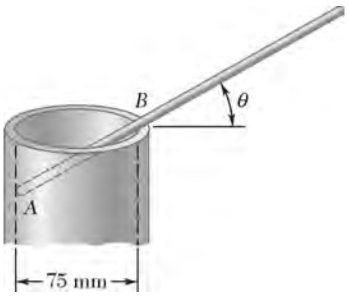
PROBLEM 8.42 (Continued)

For $N_A = 0$: $\circlearrowright \Sigma M_B = 0$: $P(0.1\text{ m}) - W(0.09326\text{ m}) = 0$

$$P = (78.48\text{ N})(0.09326\text{ m})/(0.1) = 73.19$$

System rotates about B: $P = 73.2\text{ N}$ ◀

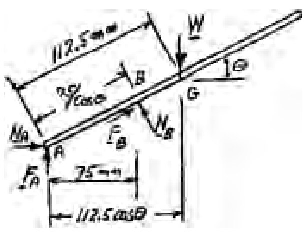
PROBLEM 8.43



A slender steel rod of length 225 mm is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

SOLUTION

Motion of rod impends down at A and to left at B.



$$F_A = \mu_s N_A \quad F_B = \mu_s N_B$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B \sin\theta + F_B \cos\theta = 0$$

$$N_A - N_B \sin\theta + \mu_s N_B \cos\theta = 0$$

$$N_A = N_B(\sin\theta - \mu_s \cos\theta) \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + N_B \cos\theta + F_B \sin\theta - W = 0$$

$$\mu_s N_A + N_B \cos\theta + \mu_s N_B \sin\theta - W = 0 \tag{2}$$

Substitute for N_A from Eq. (1) into Eq. (2):

$$\mu_s N_B(\sin\theta - \mu_s \cos\theta) + N_B \cos\theta + \mu_s N_B \sin\theta - W = 0$$

$$N_B = \frac{W}{(1-\mu_s^2)\cos\theta + 2\mu_s \sin\theta} = \frac{W}{(1-0.2^2)\cos\theta + 2(0.2)\sin\theta} \tag{3}$$

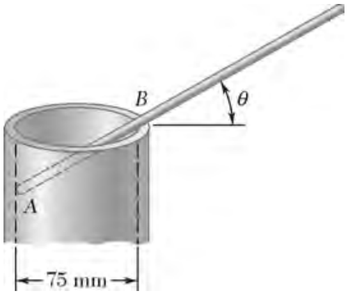
$$+\circlearrowleft \Sigma M_A = 0: \quad N_B\left(\frac{75}{\cos\theta}\right) - W(112.5\cos\theta) = 0$$

Substitute for N_B from Eq. (3), cancel W, and simplify to find

$$9.6\cos^3\theta + 4\sin\theta\cos^2\theta - 6.6667 = 0$$

$$\cos^3\theta(2.4 + \tan\theta) = 1.6667$$

Solve by trial & error: $\theta = 35.8° \blacktriangleleft$



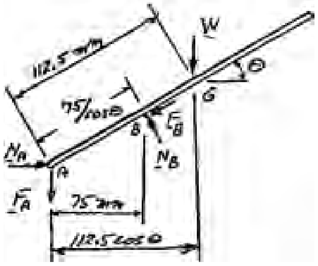
PROBLEM 8.44

In Problem 8.43, determine the smallest value of θ for which the rod will not fall out the pipe.

PROBLEM 8.43 A slender steel rod of length 225 mm is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

SOLUTION

<u>Motion of rod impends up at *A* and right at *B*.</u>



$$F_A = \mu_s N_A \qquad F_B = \mu_s N_B$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B \sin\theta - F_B \cos\theta = 0$$

$$N_A - N_B \sin\theta - \mu_s N_B \cos\theta = 0$$

$$N_A = N_B(\sin\theta + \mu_s \cos\theta) \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad -F_A + N_B \cos\theta - F_B \sin\theta - W = 0$$

$$-\mu_s N_A + N_B \cos\theta - \mu_s N_B \sin\theta - W = 0 \tag{2}$$

Substitute for N_A from Eq. (1) into Eq. (2):

$$-\mu_s N_B(\sin\theta + \mu_s \cos\theta) + N_B \cos\theta - \mu_s N_B \sin\theta - W = 0$$

$$N_B = \frac{W}{(1-\mu_s^2)\cos\theta - 2\mu_s \sin\theta} = \frac{W}{(1-0.2^2)\cos\theta - 2(0.2)\sin\theta} \tag{3}$$

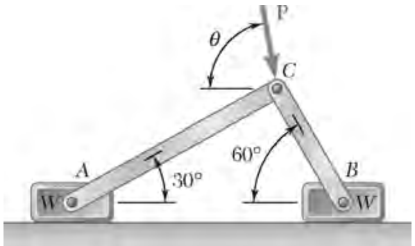
$$+\circlearrowleft \Sigma M_A = 0: \quad N_B\left(\frac{75}{\cos\theta}\right) - W(112.5\cos\theta) = 0$$

Substitute for N_B from Eq. (3), cancel W, and simplify to find

$$9.6\cos^3\theta - 4\sin\theta\cos^2\theta - 6.6667 = 0$$

$$\cos^3\theta(2.4 - \tan\theta) = 1.6667$$

Solve by trial + error: $\theta = 20.5° \blacktriangleleft$

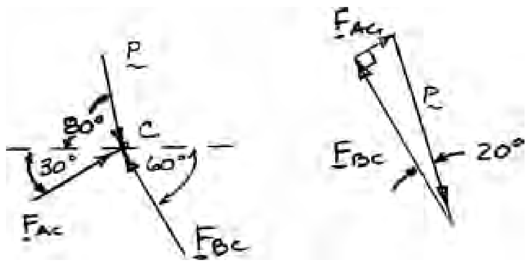


PROBLEM 8.45

Two slender rods of negligible weight are pin-connected at C and attached to blocks A and B, each of weight W. Knowing that $\theta = 80°$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD pin *C*:



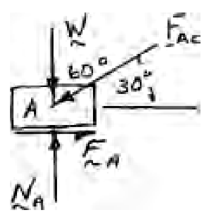
$$F_{AC} = P \sin 20° = 0.34202P$$

$$F_{BC} = P \cos 20° = 0.93969P$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W - F_{AC} \sin 30° = 0$$

or $$N_A = W + 0.34202P \sin 30° = W + 0.171010P$$

FBD block *A*: $$\Sigma F_x = 0: \quad F_A - F_{AC} \cos 30° = 0$$



or $$F_A = 0.34202P \cos 30° = 0.29620P$$

For impending motion at A: $$F_A = \mu_s N_A$$

Then $$N_A = \frac{F_A}{\mu_s}: \quad W + 0.171010P = \frac{0.29620}{0.3} P$$

or $$P = 1.22500W$$

$$\uparrow \Sigma F_y = 0: \quad N_B - W - F_{BC} \cos 30° = 0$$

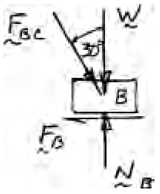
$$N_B = W + 0.93969P \cos 30° = W + 0.81380P$$

$$\Sigma F_x = 0: \quad F_{BC} \sin 30° - F_B = 0$$

$$F_B = 0.93969P \sin 30° = 0.46985P$$

PROBLEM 8.45 (Continued)

FBD block *B*:



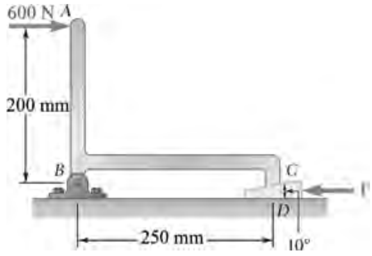
For impending motion at *B*: $F_B = \mu_s N_B$

Then $N_B = \dfrac{F_B}{\mu_s}$: $W + 0.81380P = \dfrac{0.46985P}{0.3}$

or $P = 1.32914W$

Thus, maximum *P* for equilibrium $P_{\max} = 1.225W$ ◀

PROBLEM 8.46



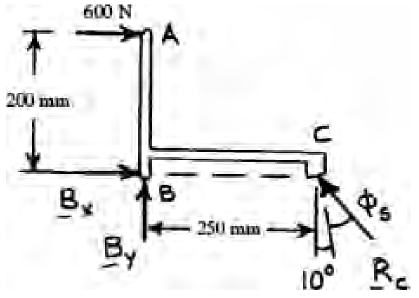
The machine part *ABC* is supported by a frictionless hinge at *B* and a 10° wedge at *C*. Knowing that the coefficient of static friction is 0.20 at both surfaces of the wedge, determine (*a*) the force **P** required to move the wedge to the left, (*b*) the components of the corresponding reaction at *B*.

SOLUTION

$$\mu_s = 0.20$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.20 = 11.3099°$$

Free body: *ABC*

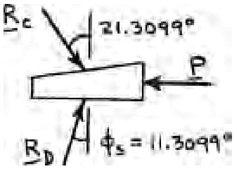


$$10° + 11.3099° = 21.3099°$$

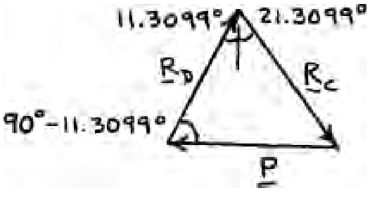
$$+\circlearrowleft \Sigma M_B = 0: \quad (R_C \cos 21.3099°)(250) - (600\text{ N})(200) = 0$$

$$R_C = 515.227\text{ N}$$

Free body: Wedge



Force triangle:



PROBLEM 8.46 (Continued)

Law of sines:

$$\frac{P}{\sin 32.6198^\circ} = \frac{(R_C = 515.227\text{ N})}{\sin 78.690^\circ} \Rightarrow P = 283.240\text{ N}$$

(*a*) **P** = 283 N ◀

(*b*) Returning to free body of *ABC*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 600 - (515.227)\sin 21.3099^\circ = 0$$

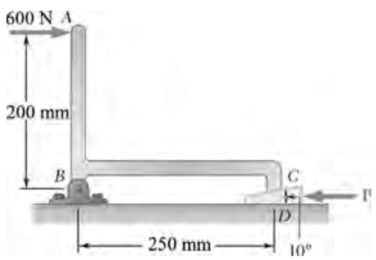
$$B_x = -412.76\text{ N}$$

$\mathbf{B}_x = 413$ N ◀

$$+\uparrow \Sigma F_y = 0: \quad B_y + (515.227)\cos 21.3099^\circ = 0$$

$$B_y = -480\text{ N}$$

$\mathbf{B}_y = 480$ N ◀



PROBLEM 8.47

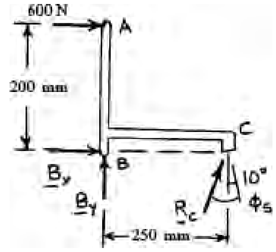
Solve Problem 8.46 assuming that the wedge is to be moved to the right.

PROBLEM 8.46 The machine part *ABC* is supported by a frictionless hinge at *B* and a 10° wedge at *C*. Knowing that the coefficient of static friction is 0.20 at both surfaces of the wedge, determine (*a*) the force **P** required to move the wedge to the left, (*b*) the components of the corresponding reaction at *B*.

SOLUTION

$$\mu_s = 0.20$$
$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.20 = 11.30993°$$

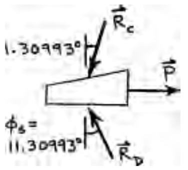
Free body: *ABC*



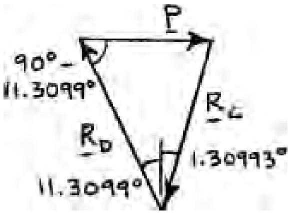
$$11.30993° - 10° = 1.30993°$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (R_C \cos 1.30993°)(250) - (600\text{ N})(200) = 0$$
$$R_C = 480.125\text{ N}$$

Free body: Wedge



Force triangle:



PROBLEM 8.47 (Continued)

Law of sines:

$$\frac{P}{\sin 12.6198^\circ} = \frac{(R_C = 480.125 \text{ N})}{\sin 78.690^\circ} \Rightarrow P = 106.975 \text{ N}$$

(*a*) $\mathbf{P} = 107 \text{ N}$ ◀

(*b*) Returning to free body of *ABC*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 600 + (480.125)\sin 1.30993^\circ = 0$$

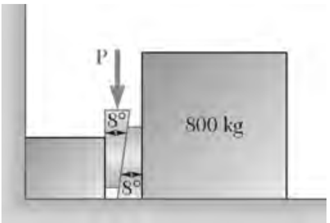
$$B_x = -610.976 \text{ N}$$

$\mathbf{B}_x = 611 \text{ N}$ ◀

$$+\uparrow \Sigma F_y = 0: \quad B_y + (480.125)\cos 1.30993^\circ = 0$$

$$B_y = -480 \text{ N}$$

$\mathbf{B}_y = 480 \text{ N}$ ◀



PROBLEM 8.48

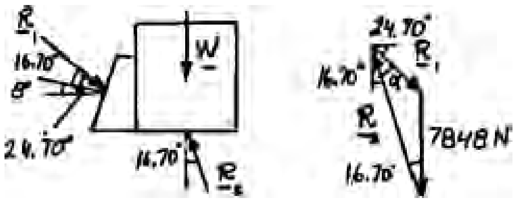
Two 8° wedges of negligible weight are used to move and position the 800-kg block. Knowing that the coefficient of static friction is 0.30 at all surfaces of contact, determine the smallest force **P** that should be applied as shown to one of the wedges.

SOLUTION

$$\mu_s = 0.30 \quad \phi_s = \tan^{-1} 0.30 = 16.70°$$

Free body: 800-kg block and right-hand wedge

$$W = (800 \text{ kg})(9.81 \text{ m/s}^2) = 7848 \text{ N}$$



Force triangle:

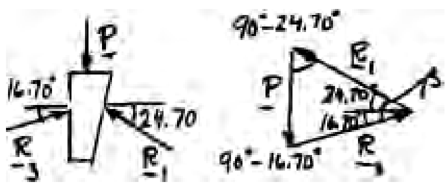
$$\alpha = 90° - 16.70° - 24.70° = 48.60°$$

Law of sines:

$$\frac{R_1}{\sin 16.70°} = \frac{7848 \text{ N}}{\sin 48.60°}$$

$$R_1 = 3006.5 \text{ N}$$

Free body: Left-hand wedge



Force triangle:

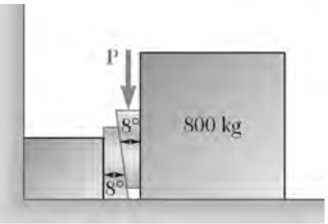
$$\beta = 16.70° + 24.70° = 41.40°$$

Law of sines:

$$\frac{P}{\sin 41.40°} = \frac{R_1}{\sin(90° - 16.70°)}$$

$$\frac{P}{\sin 41.40°} = \frac{3006.5 \text{ N}}{\cos 16.70°}$$

P = 2080 N ◀



PROBLEM 8.49

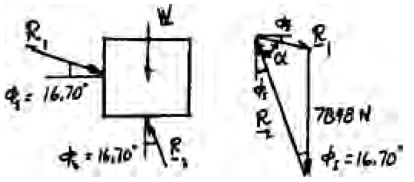
Two 8° wedges of negligible weight are used to move and position the 800-kg block. Knowing that the coefficient of static friction is 0.30 at all surfaces of contact, determine the smallest force **P** that should be applied as shown to one of the wedges.

SOLUTION

Solution

$$\mu_s = 0.30 \quad \phi_s = \tan^{-1} 0.30 = 16.70°$$

Free body: 800-kg block



Force triangle:

$$W = (800\text{ kg})(9.81\text{ m/s}^2) = 7848\text{ N}$$

$$\alpha = 90° - 2\phi_s = 90° - 2(16.70°)$$
$$= 56.60°$$

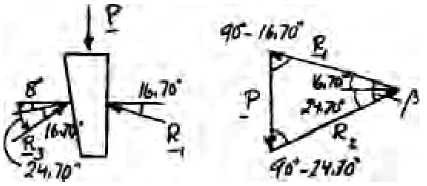
Law of sines:

$$\frac{R_1}{\sin 16.70°} = \frac{7848\text{ N}}{\sin 56.60°}$$

$$R_1 = 2701\text{ N}$$

Free body: Right-hand wedge

Force triangle:



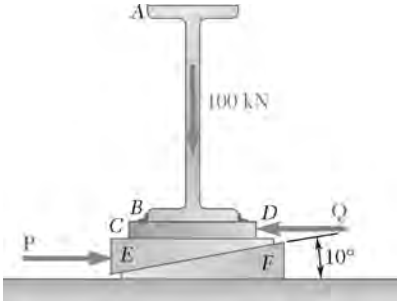
$$\beta = 16.70° + 24.70°$$
$$= 41.40°$$

Law of sines:

$$\frac{P}{\sin 41.40°} = \frac{R_1}{\sin(90° - 24.70°)}$$

$$\frac{P}{\sin 41.40°} = \frac{2701\text{ N}}{\cos 24.70}$$

$\mathbf{P} = 1966\text{ N}$ ◄

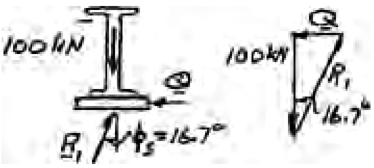


PROBLEM 8.50

The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 100 kN. The coefficient of static friction is 0.30 between two steel surfaces and 0.60 between steel and concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

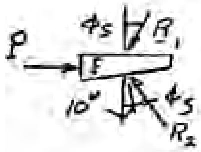
SOLUTION

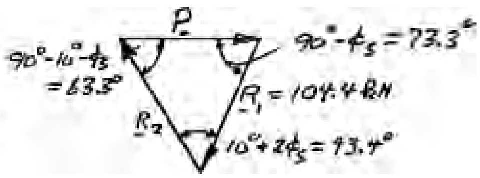
Free body: Beam and plate *CD*



$$R_1 = \frac{(100 \text{ kN})}{\cos 16.7}$$

$$R_1 = 104.4 \text{ kN}$$





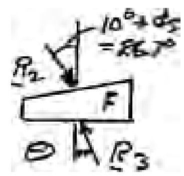
(*a*) $$\frac{P}{\sin 43.4°} = \frac{104.4 \text{ kN}}{\sin 63.3°}$$ $\mathbf{P} = 80.3$ kN ◀

(*b*) $$\phi_s = \tan^{-1} 0.3 = 16.7°$$

$$Q = (100 \text{ kN}) \tan 16.7°$$ $\mathbf{Q} = 30$ kN ◀

Free body: Wedge *F*

(To check that it does not move.)

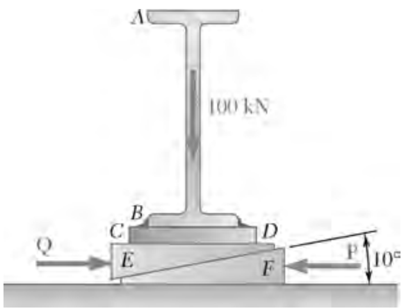


Since wedge *F* is a two-force body, R_2 and R_3 are colinear

Thus $$\theta = 26.7°$$

But $$\phi_{\text{concrete}} = \tan^{-1} 0.6 = 31.0° > \theta \textbf{ OK}$$

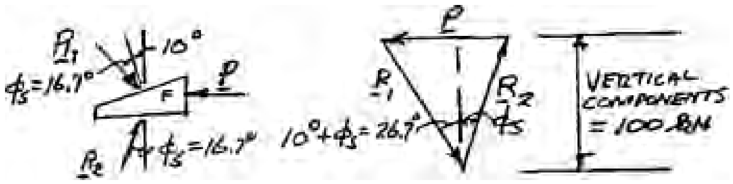
PROBLEM 8.51



The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges E and F. The base plate CD has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 100 kN. The coefficient of static friction is 0.30 between two steel surfaces and 0.60 between steel and concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

SOLUTION

Free body: Wedge F $\qquad \phi_s = \tan^{-1} 0.30 = 16.7°$



(*a*)

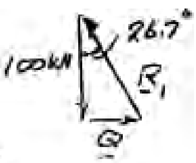
$$P = (100 \text{ kN}) \tan 26.7° + (100 \text{ kN}) \tan \phi_s$$

$$P = 50.29 \text{ kN} + 30 \text{ kN}$$

$$P = 80.29 \text{ kN} \qquad \mathbf{P} = 80.3 \text{ kN} \leftarrow \blacktriangleleft$$

$$R_1 = \frac{(100 \text{ kN})}{\cos 26.7°} = 111.94 \text{ kN}$$

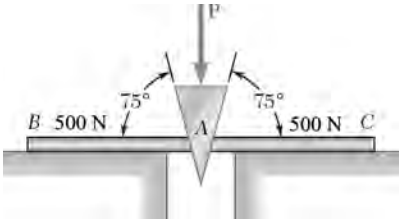
Free body: Beam, plate, and wedge E



(*b*)

$$Q = W \tan 26.7° = (100 \text{ kN}) \tan 26.7°$$

$$Q = 50.29 \text{ kN} \qquad \mathbf{Q} = 50.3 \text{ kN} \rightarrow \blacktriangleleft$$



PROBLEM 8.52

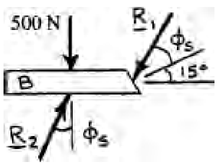
A wedge A of negligible weight is to be driven between two 500 N plates B and C. The coefficient of static friction between all surfaces of contact is 0.35. Determine the magnitude of the force **P** required to start moving the wedge (*a*) if the plates are equally free to move, (*b*) if plate C is securely bolted to the surface.

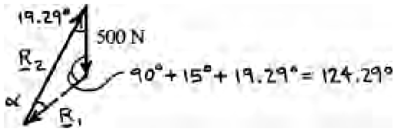
SOLUTION

(*a*) <u>With plates equally free to move</u>

<u>Free body: Plate B</u> $\qquad \phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.35 = 19.2900°$

Force triangle:





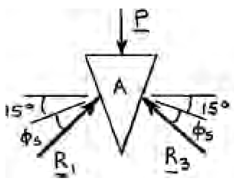
$$\alpha = 180° - 124.29° - 19.29° = 36.42°$$

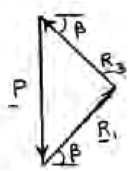
Law of sines: $$\frac{R_1}{\sin 19.29°} = \frac{500\text{ N}}{\sin 36.42°}$$

$$R_1 = 278.213\text{ N}$$

<u>Free body: Wedge A</u>

Force triangle:





By symmetry, $\qquad R_3 = R_1 = 278.213\text{ N}$

$$\beta = 19.29° + 15° = 34.29°$$

Then $\qquad P = 2R_1 \sin\beta$

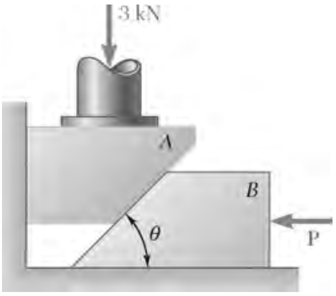
or $\qquad P = 2(278.213)\sin 34.29°$

$$= 313.48\text{ N}$$

$P = 313\text{ N}$ ◀

(*b*) <u>With plate C bolted</u>

The free body diagrams of plate B and wedge A (the only members to move) are same as above. Answer is thus the same. $\qquad P = 313\text{ N}$ ◀



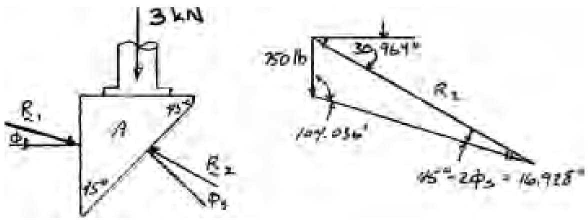
PROBLEM 8.53

Block *A* supports a pipe column and rests as shown on wedge *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** required to raise block *A*.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.25 = 14.036°$$

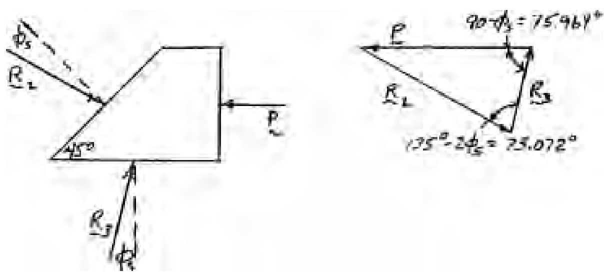
FBD block *A*:



$$\frac{R_2}{\sin 104.036°} = \frac{3\text{ kN}}{\sin 16.928°}$$

$$R_2 = 10.0000\text{ kN}$$

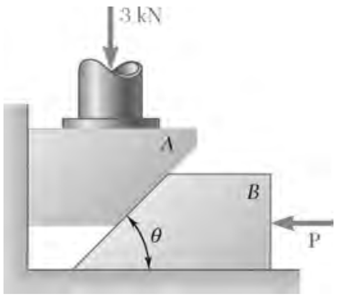
FBD wedge *B*:



$$\frac{P}{\sin 73.072°} = \frac{10.0000\text{ kN}}{\sin 75.964°}$$

$$P = 9.8611\text{ kN}$$

$\mathbf{P} = 9.86\text{ kN} \leftarrow$ ◀



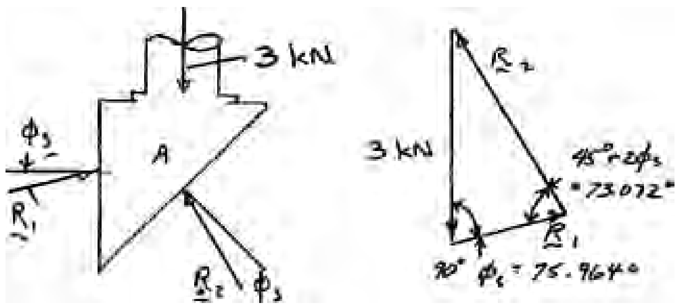
PROBLEM 8.54

Block A supports a pipe column and rests as shown on wedge B. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** for which equilibrium is maintained.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.25 = 14.036°$$

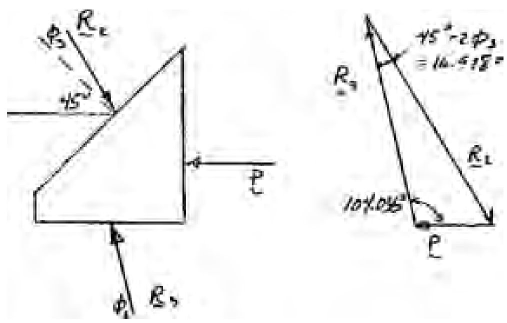
FBD block *A*:



$$\frac{R_2}{\sin(75.964°)} = \frac{3\text{ kN}}{\sin(73.072°)}$$

$$R_2 = 3.0420\text{ kN}$$

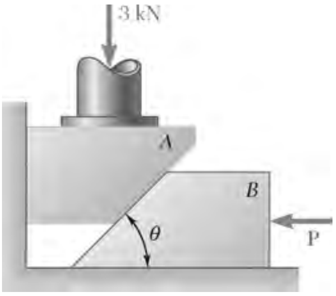
FBD wedge *B*:



$$\frac{P}{\sin 16.928°} = \frac{3.0420\text{ kN}}{\sin 104.036°}$$

$$P = 0.91300\text{ kN}$$

P = 913 N ← ◀

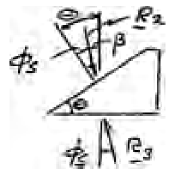


PROBLEM 8.55

Block A supports a pipe column and rests as shown on wedge B. The coefficient of static friction at all surfaces of contact is 0.25. If $\mathbf{P} = 0$, determine (a) the angle θ for which sliding is impending, (b) the corresponding force exerted on the block by the vertical wall.

SOLUTION

Free body: Wedge B $\qquad \phi_s = \tan^{-1} 0.25 = 14.04°$



(a) Since wedge is a two-force body, $\mathbf{R}_2$ and $\mathbf{R}_3$ must be equal and opposite. Therefore, they form equal angles with vertical

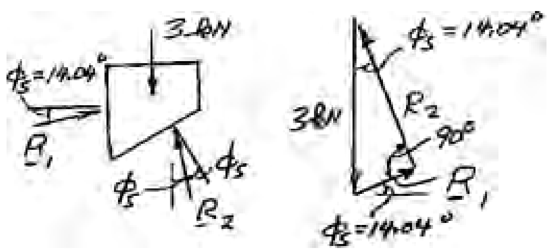
$$\beta = \phi_s$$

and

$$\theta - \phi_s = \phi_s$$
$$\theta = 2\phi_s = 2(14.04°)$$

$$\theta = 28.1° \blacktriangleleft$$

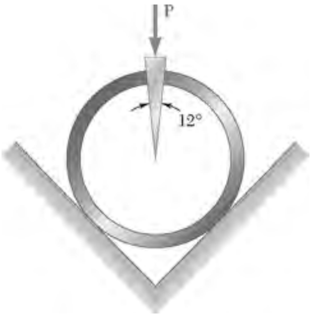
Free body: Block A



$$R_1 = (3 \text{ kN}) \sin 14.04° = 0.7278 \text{ kN}$$

(b) Force exerted by wall: $\qquad \mathbf{R}_1 = 728$ N $\angle$ 14.04° ◀

PROBLEM 8.56

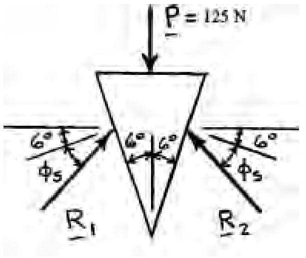


A 12° wedge is used to spread a split ring. The coefficient of static friction between the wedge and the ring is 0.30. Knowing that a force **P** of magnitude 125 N was required to insert the wedge, determine the magnitude of the forces exerted on the ring by the wedge after insertion.

SOLUTION

Free body: Wedge

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.30 = 16.6992°$$

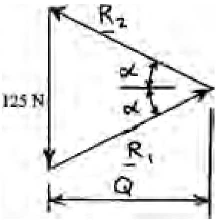


Force triangle:

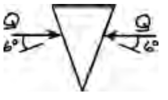
$$\alpha = 6° + \phi_s$$
$$= 6° + 16.6992°$$
$$= 22.6992°$$

Q = Horizontal component of R

$$Q = \frac{\frac{1}{2}(125\text{ N})}{\tan 22.6992°}$$
$$= 149.417\text{ N}$$



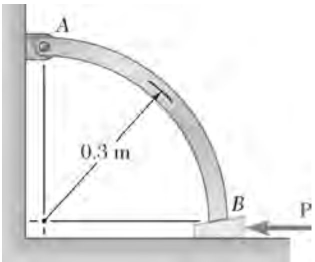
Free body: After wedge has been inserted



Wedge is now a two-force body with forces shown.

$Q = 149.4\text{ N}$ ◄

(*Note:* Since angles between force **Q** and normal to wedge is $6° < \phi_s$, wedge stays in place.)

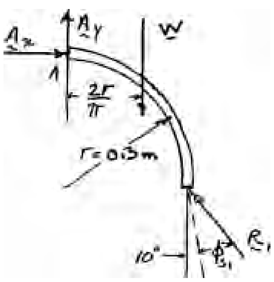


PROBLEM 8.57

A 10° wedge is to be forced under end *B* of the 5-kg rod *AB*. Knowing that the coefficient of static friction is 0.40 between the wedge and the rod and 0.20 between the wedge and the floor, determine the smallest force **P** required to raise end *B* of the rod.

SOLUTION

FBD *AB*:



$$W = mg$$

$$W = (5\text{ kg})(9.81\text{ m/s}^2)$$

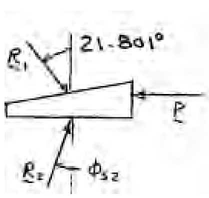
$$W = 49.050\text{ N}$$

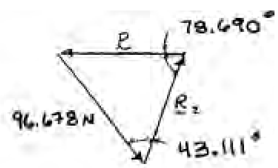
$$\phi_{s1} = \tan^{-1}(\mu_s)_1 = \tan^{-1} 0.40 = 21.801°$$

$$\circlearrowleft \Sigma M_A = 0: \quad rR_1\cos(10° + 21.801°) - rR_1\sin(10° + 21.801°) - \frac{2r}{\pi}(49.050\text{ N}) = 0$$

$$R_1 = 96.678\text{ N}$$

FBD wedge:



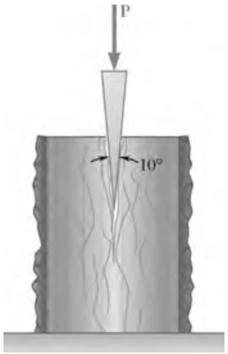


$$\phi_{s2} = \tan^{-1}(\mu_s)_2 = \tan^{-1} 0.20 = 11.3099$$

$$\frac{P}{\sin(43.111°)} = \frac{96.678\text{ N}}{\sin 78.690°}$$

P = 67.4 N ← ◀

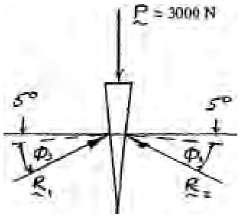
PROBLEM 8.58



A 10° wedge is used to split a section of a log. The coefficient of static friction between the wedge and the log is 0.35. Knowing that a force **P** of magnitude 3000 N was required to insert the wedge, determine the magnitude of the forces exerted on the wood by the wedge after insertion.

SOLUTION

FBD wedge (impending motion ↓):



$$\phi_s = \tan^{-1}\mu_s$$
$$= \tan^{-1} 0.35$$
$$= 19.29°$$

By symmetry: $R_1 = R_2$

$$\uparrow \Sigma F_y = 0: \quad 2R_1 \sin(5° + \phi_s) - 3000 \text{ N} = 0$$

or
$$R_1 = R_2 = \frac{1500 \text{ N}}{\sin(5° + 19.29°)} = 3646.48 \text{ N}$$

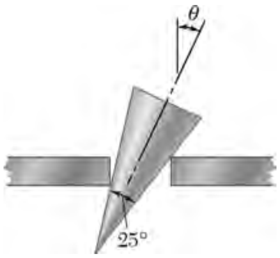
When P is removed, the vertical components of R_1 and R_2 vanish, leaving the horizontal components

$$R_{1x} = R_{2x}$$
$$= R_1 \cos(5° + \phi_s)$$
$$= (3646.48)\cos(5° + 19.29°)$$
$$= 3323.68 \text{ N}$$

$R_{1x} = R_{2x} = 3320 \text{ N}$ ◀

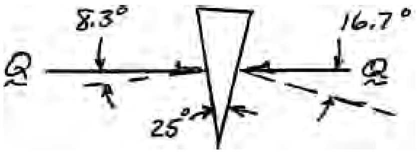
(Note that $\phi_s > 5°$, so wedge is self-locking.)

PROBLEM 8.59



A conical wedge is placed between two horizontal plates that are then slowly moved toward each other. Indicate what will happen to the wedge (*a*) if $\mu_s = 0.20$, (*b*) if $\mu_s = 0.30$.

SOLUTION



As the plates are moved, the angle θ will decrease.

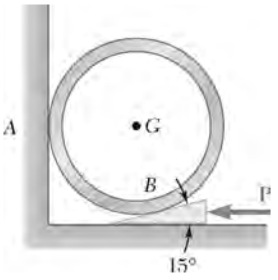
(*a*) $\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.2 = 11.31°.$

As θ decrease, the minimum angle at the contact approaches

$12.5° > \phi_s = 11.31°,$ so the wedge will slide up and out from the slot. ◀

(*b*) $\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.3 = 16.70°.$

As θ decreases, the angle at one contact reaches 16.7°. (At this time the angle at the other contact is $25° - 16.7° = 8.3° < \phi_s$). The wedge binds in the slot. ◀

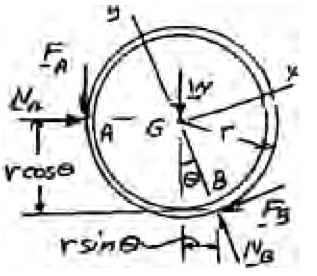


PROBLEM 8.60

A 15° wedge is forced under a 50-kg pipe as shown. The coefficient of static friction at all surfaces is 0.20. (*a*) Show that slipping will occur between the pipe and the vertical wall. (*b*) Determine the force **P** required to move the wedge.

SOLUTION

Free body: Pipe



$$+\circlearrowleft \Sigma M_B = 0: \quad Wr\sin\theta + F_A r(1+\sin\theta) - N_A r\cos\theta = 0$$

Assume slipping at A:

$$F_A = \mu_s N_A$$

$$N_A\cos\theta - \mu_s N_A(1+\sin\theta) = W\sin\theta$$

$$N_A = \frac{W\sin\theta}{\cos\theta - \mu_s(1+\sin\theta)}$$

$$N_A = \frac{W\sin 15^\circ}{\cos 15^\circ - (0.20)(1+\sin 15^\circ)}$$

$$= 0.36241W$$

$$+\nearrow \Sigma F_x = 0: \quad -F_B - W\sin\theta - F_A\sin\theta + N_A\cos\theta = 0$$

$$F_B = N_A\cos\theta - \mu_s N_A\sin\theta - W\sin\theta$$

$$F_B = 0.36241W\cos 15^\circ - 0.20(0.36241W)\sin 15^\circ - W\sin 15^\circ$$

$$F_B = 0.072482W$$

$$+\nwarrow \Sigma F_y = 0: \quad N_B - W\cos\theta - F_A\cos\theta - N_A\sin\theta = 0$$

$$N_B = N_A\sin\theta + \mu_s N_A\cos\theta + W\cos\theta$$

$$N_B = (0.36241W)\sin 15^\circ + 0.20(0.36241W)\cos 15^\circ + W\cos 15^\circ$$

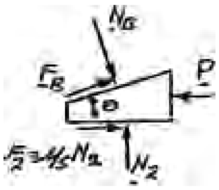
$$N_B = 1.12974W$$

Maximum available: $F_B = \mu_s N_B = 0.22595W$

(*a*) We note that $F_B < F_{\max}$ — No slip at B ◀

PROBLEM 8.60 (Continued)

(*b*) Free body: Wedge



$$+\uparrow \Sigma F_y = 0: \quad N_2 - N_B \cos\theta + F_B \sin\theta = 0$$

$$N_2 = N_B \cos\theta - F_B \sin\theta$$

$$N_2 = (1.12974W)\cos 15° - (0.07248W)\sin 15°$$

$$N_2 = 1.07249W$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_B \cos\theta + N_B \sin\theta + \mu_s N_2 - P = 0$$

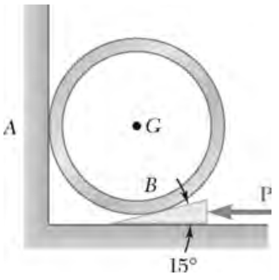
$$P = F_B \cos\theta + N_B \sin\theta + \mu_s N_2$$

$$P = (0.07248W)\cos 15° + (1.12974W)\sin 15° + 0.2(1.07249W)$$

$$P = 0.5769W$$

$$W = mg: \quad P = 0.5769(50 \text{ kg})(9.81 \text{ m/s}^2)$$

P = 283 N ← ◀

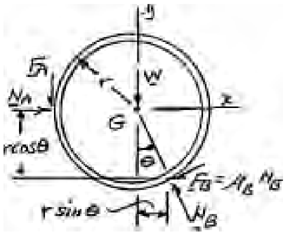


PROBLEM 8.61

A 15° wedge is forced under a 50-kg pipe as shown. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine the largest coefficient of static friction between the pipe and the vertical wall for which slipping will occur at A.

SOLUTION

<u>Free body: Pipe</u>



$$\circlearrowleft^{+}\ \Sigma M_A = 0: \quad N_B r\cos\theta - \mu_B N_B r - (\mu_B N_B \sin\theta)r - Wr = 0$$

$$N_B = \frac{W}{\cos\theta - \mu_B(1+\sin\theta)}$$

$$N_B = \frac{W}{\cos 15° - 0.2(1+\sin 15°)}$$

$$N_B = 1.4002W$$

$$\xrightarrow{+}\ \Sigma F_x = 0: \quad N_A - N_B \sin\theta - \mu_B N_B \cos\theta = 0$$

$$N_A = N_B(\sin\theta + \mu_B \cos\theta)$$

$$= (1.4002W)(\sin 15° + 0.2\times\cos 15°)$$

$$N_A = 0.63293W$$

$$+\uparrow\ \Sigma F_y = 0: \quad -F_A - W + N_B\cos\theta - \mu_B N_B \sin\theta = 0$$

$$F_A = N_B(\cos\theta - \mu_B\sin\theta) - W$$

$$F_A = (1.4002W)(\cos 15° - 0.2\times\sin\theta) - W$$

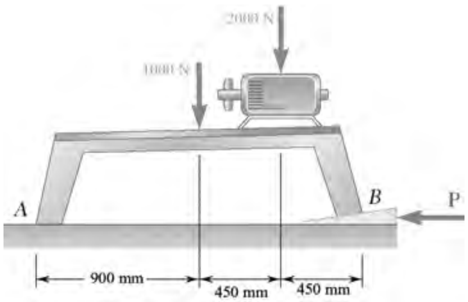
$$F_A = 0.28001W$$

For slipping at A:

$$F_A = \mu_A N_A$$

$$\mu_A = \frac{F_A}{N_A} = \frac{0.28001W}{0.63293W}$$

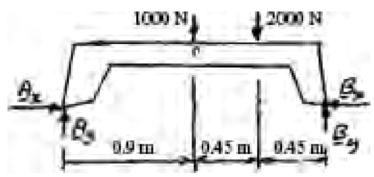
$\mu_A = 0.442$ ◀



PROBLEM 8.62

An 8° wedge is to be forced under a machine base at *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.15, (*a*) determine the force **P** required to move the wedge, (*b*) indicate whether the machine base will slide on the floor.

SOLUTION



Free body: Machine base

$$+\circlearrowleft \Sigma M_B = 0: \quad (1000\text{ N})(0.9\text{ m}) + (2000\text{ m})(0.45\text{ m}) - A_y(1.8\text{ m}) = 0$$

$$A_y = 1000\text{ N}\uparrow$$

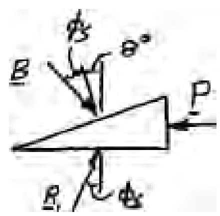
$$+\uparrow \Sigma F_y = 0: \quad A_y + B_y - 1000\text{ N} - 2000\text{ N} = 0$$

$$1000\text{ N} + B_y - 1000\text{ N} - 2000\text{ N} = 0$$

$$B_y = 2000\text{ N}\uparrow$$

Free body: Wedge

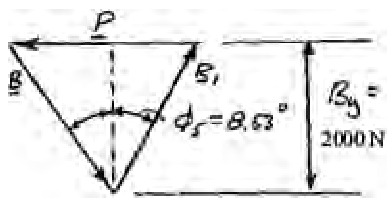
(Assume machine base will not move)



$$\mu_s = 0.15, \quad \phi_s = \tan^{-1} 0.15 = 8.53°$$

We know that $B_y = 2000\text{ N}$

Force triangle: $8° + \phi_s = 8° + 8.53° = 16.53°$



$$P = (2000\text{ N})\tan 16.53° + (2000\text{ N})\tan 8.53°$$

$$P = 893.54\text{ N}$$

Total maximum friction force at *A* and *B*:

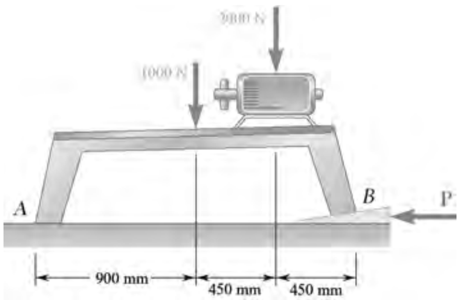
$$F_m = \mu_s W = 0.15(1000\text{ N} + 2000\text{ N}) = 450\text{ N}$$

If machine moves with wedge: $P = F_m = 450\text{ N}$

Using smaller *P*, we have

(*a*) $P = 450\text{ N}$ ◀

(*b*) Machine base moves ◀

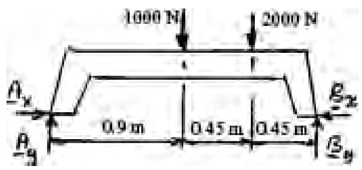


PROBLEM 8.63

Solve Problem 8.62 assuming that the wedge is to be forced under the machine base at *A* instead of *B*.

PROBLEM 8.62 An 8° wedge is to be forced under a machine base at *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.15, (*a*) determine the force **P** required to move the wedge, (*b*) indicate whether the machine base will slide on the floor.

SOLUTION



FBD: Machine base

$$\circlearrowleft + \Sigma M_B = 0: \quad (1000\text{ N})(0.9\text{ m}) + (2000\text{ N})(0.45\text{ m}) - A_y(1.8\text{ m}) = 0$$

$$\mathbf{A}_y = 1000\text{ N}\uparrow$$

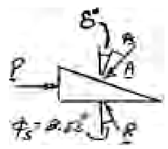
$$+\uparrow \Sigma F_y = 0: \quad A_y + B_y - 1000\text{ N} - 2000\text{ N} = 0$$

$$\mathbf{B}_y = 2000\text{ N}\uparrow$$

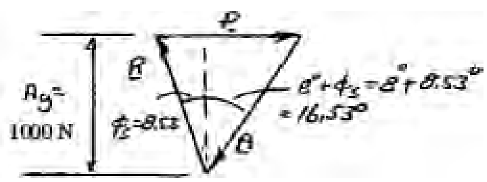
$$\phi_s = \tan^{-1} 0.15 = 8.53°$$

We known that $\quad A_y = 1000$ N

FBD: Wedge



Force triangle:

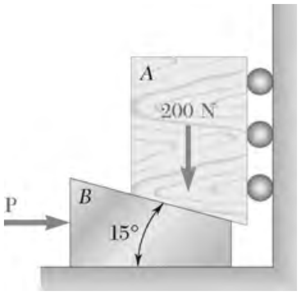


(*a*) $\quad P = (1000\text{ N})\tan 8.53° + (1000\text{ N})\tan 16.53° = 446.77\text{ N}$ $\qquad P = 447\text{ N}$ ◀

(*b*) $\quad$ Total maximum friction force at *A* and *B*:

$$F_m = \mu_s(W) = 0.15(1000\text{ N} + 2000\text{ N}) = 450\text{ N}$$

Since $P < F_m$, $\qquad$ machine base will not move ◀



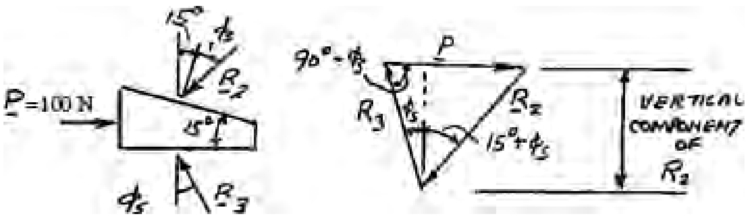
PROBLEM 8.64*

A 200-N block rests as shown on a wedge of negligible weight. The coefficient of static friction m_s is the same at both surfaces of the wedge, and friction between the block and the vertical wall may be neglected. For $P = 100$ N, determine the value of m_s for which motion is impending. (*Hint:* Solve the equation obtained by trial and error.)

SOLUTION

Free body: Wedge

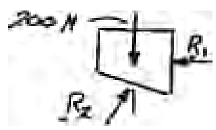
Force triangle:



Law of sines: $$\frac{R_2}{\sin(90° - \phi_s)} = \frac{P}{\sin(15° + 2\phi_s)}$$

$$R_2 = P\frac{\sin(90° - \phi_s)}{\sin(15° + 2\phi_s)} \qquad (1)$$

Free body: Block



$$\Sigma F_y = 0$$

Vertical component of R_2 is 200 N

Return to force triangle of wedge. Note $P = 100$ N

$$100 \text{ N} = (200 \text{ N})\tan\phi + (200 \text{ N})\tan(15° + \phi_s)$$

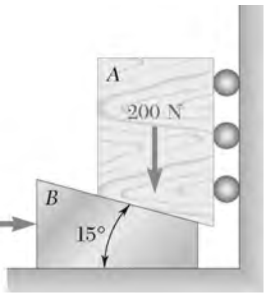
$$0.5 = \tan\phi + \tan(15° + \phi_s)$$

Solve by trial and error $\phi_s = 6.292$

$$\mu_s = \tan\phi_s = \tan 6.292°$$

$\mu_s = 0.1103$ ◀

PROBLEM 8.65*



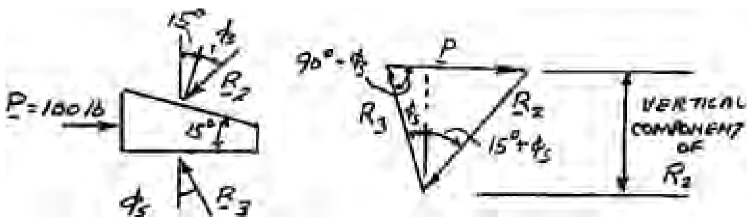
Solve Problem 8.64 assuming that the rollers are removed and that μ_s is the coefficient of friction at all surfaces of contact.

PROBLEM 8.64* A 200-N block rests as shown on a wedge of negligible weight. The coefficient of static friction μ_s is the same at both surfaces of the wedge, and friction between the block and the vertical wall may be neglected. For P = 100 N, determine the value of μ_s for which motion is impending. (*Hint:* Solve the equation obtained by trial and error.)

SOLUTION

Free body: Wedge

Force triangle:

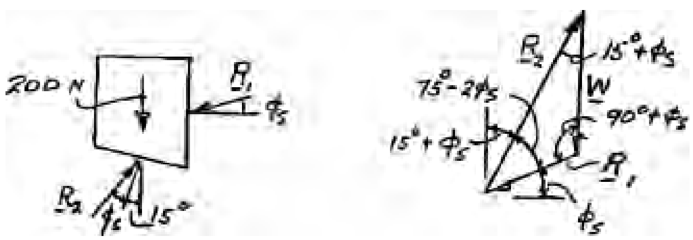


Law of sines:
$$\frac{R_2}{\sin(90° - \phi_s)} = \frac{P}{\sin(15° + 2\phi_s)}$$

$$R_2 = P\frac{\sin(90° - \phi_s)}{\sin(15° + 2\phi_s)} \qquad (1)$$

Free body: Block (Rollers removed)

Force triangle:



Law of sines:
$$\frac{R_2}{\sin(90° + \phi_s)} = \frac{W}{\sin(75° - 2\phi_s)}$$

$$R_2 = W\frac{\sin(90° + \phi_s)}{\sin(75° - 2\theta_s)} \qquad (2)$$

PROBLEM 8.65* (Continued)

Equate R_2 from Eq. (1) and Eq. (2):

$$P\frac{\sin(90°-\phi_s)}{\sin(15°+2\phi_s)} = W\frac{\sin(90°+\phi_s)}{\sin(75°-2\phi_s)}$$

$$P = 100 \text{ N}$$

$$W = 200 \text{ N}$$

$$0.5 = \frac{\sin(90°+\phi_s)\sin(15°+2\phi_s)}{\sin(75°-2\phi_s)\sin(90°-\phi_s)}$$

Solve by trial and error: $\phi_s = 5.784°$

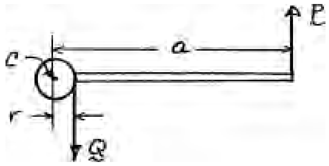
$$\mu_s = \tan\phi_s = \tan 5.784°$$

$\mu_s = 0.1013$ ◀

PROBLEM 8.66

Derive the following formulas relating the load **W** and the force **P** exerted on the handle of the jack discussed in Section 8.6. (*a*) $P = (Wr/a)\tan(\theta + \phi_s)$, to raise the load; (*b*) $P = (Wr/a)\tan(\phi_s \quad \theta)$, to lower the load if the screw is self-locking; (*c*) $P = (Wr/a)\tan(\theta \quad \phi_s)$, to hold the load if the screw is not self-locking.

SOLUTION

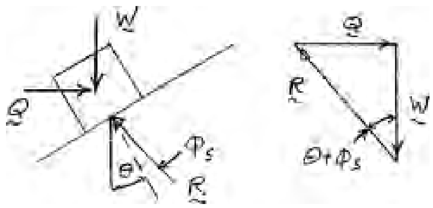


FBD jack handle:

See Section 8.6. $\circlearrowleft \Sigma M_C = 0: \quad aP - rQ = 0 \quad \text{or} \quad P = \frac{r}{a}Q$

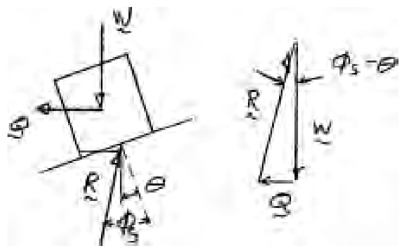
FBD block on incline:

(*a*) Raising load



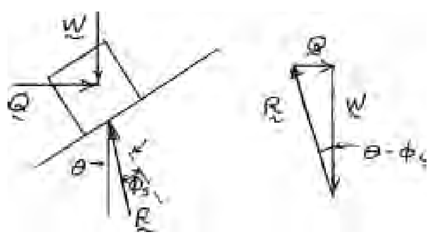
$$Q = W\tan(\theta + \phi_s) \qquad\qquad P = \frac{r}{a}W\tan(\theta + \phi_s) \blacktriangleleft$$

(*b*) Lowering load if screw is self-locking (i.e.: if $\phi_s > \theta$)

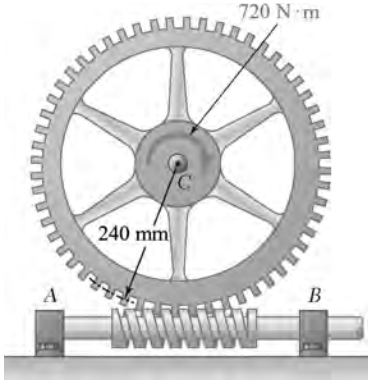


$$Q = W\tan(\phi_s - \theta) \qquad\qquad P = \frac{r}{a}W\tan(\phi_s - \theta) \blacktriangleleft$$

(*c*) Holding load if screw is not self-locking (i.e., if $\phi_s < \theta$)



$$Q = W\tan(\theta - \phi_s) \qquad\qquad P = \frac{r}{a}W\tan(\theta - \phi_s) \blacktriangleleft$$

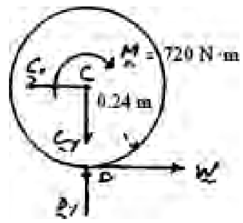


PROBLEM 8.67

The square-threaded worm gear shown has a mean radius of 30 mm and a lead of 7.5 mm. The large gear is subjected to a constant clockwise couple of 720 N · m. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft *AB* in order to rotate the large gear counterclockwise. Neglect friction in the bearings at *A*, *B*, and *C*.

SOLUTION

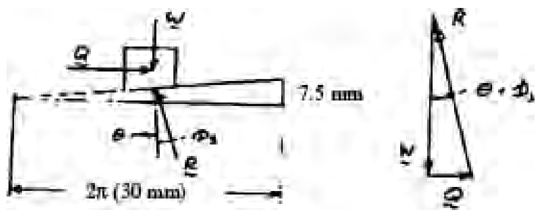
FBD large gear:



$$\circlearrowleft \Sigma M_C = 0: \quad (0.24 \text{ m})W - 720 \text{ N} \cdot \text{m} = 0$$

$$W = 3000 \text{ N}$$

Block on incline:

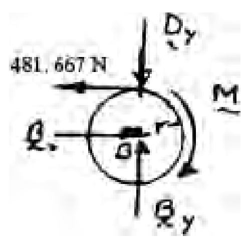


$$\theta = \tan^{-1} \frac{7.5 \text{ mm}}{2\pi(30 \text{ mm})} = 2.2785°$$

$$\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.12$$
$$= 6.8428°$$

$$Q = W \tan(\theta + \phi_s)$$
$$= (3000 \text{ N}) \tan 9.1213°$$
$$= 481.667 \text{ N}$$

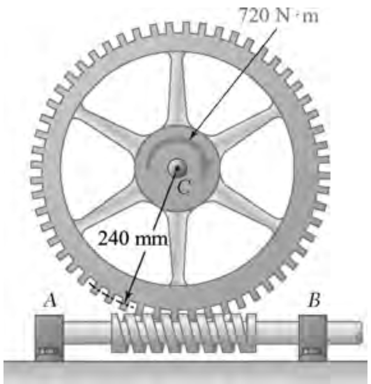
FBD worm gear:



$$r = 30 \text{ mm} = 0.03 \text{ m}$$

$$\circlearrowleft \Sigma M_B = 0: \quad (0.03 \text{ m})(481.667 \text{ N}) - M = 0$$

$$M = 14.45 \text{ N} \cdot \text{m} \blacktriangleleft$$



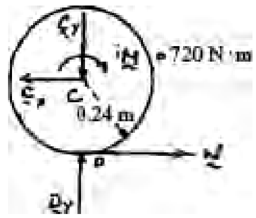
PROBLEM 8.68

In Problem 8.67, determine the couple that must be applied to shaft *AB* in order to rotate the large gear clockwise.

PROBLEM 8.67 The square-threaded worm gear shown has a mean radius of 30 mm and a lead of 7.5 mm. The large gear is subjected to a constant clockwise couple of 720 N · m. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft *AB* in order to rotate the large gear counterclockwise. Neglect friction in the bearings at *A*, *B*, and *C*.

SOLUTION

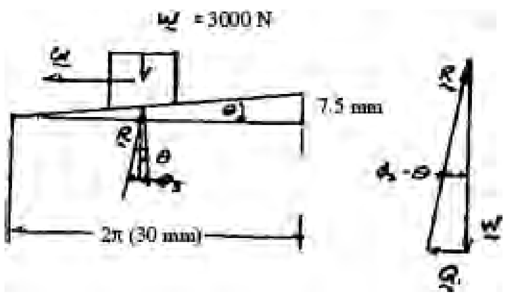
FBD large gear:



$$\circlearrowleft \Sigma M_C = 0: \quad (0.24 \text{ m})W - 720 \text{ N} \cdot \text{m} = 0$$

$$W = 3000 \text{ N}$$

Block on incline:



$$\theta = \tan^{-1}\frac{7.5 \text{ mm}}{2\pi(30 \text{ mm})}$$

$$= 2.2785°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.12$$

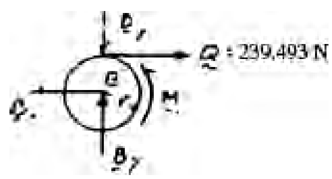
$$= 6.8428°$$

$$Q = W\tan(\phi_s - \theta)$$

$$= (3000 \text{ N})\tan 4.5643°$$

$$= 239.493 \text{ N}$$

FBD worm gear:



$$r = 30 \text{ mm} = 0.03 \text{ m}$$

$$\circlearrowleft \Sigma M_B = 0: \quad M - (0.03 \text{ m})(239.493 \text{ N}) = 0$$

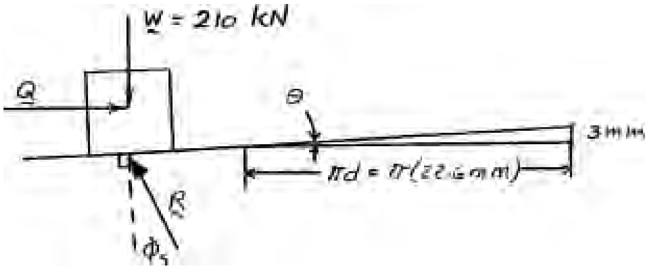
$$M = 7.18 \text{ N} \cdot \text{m} \blacktriangleleft$$

PROBLEM 8.69

High-strength bolts are used in the construction of many steel structures. For a 24-mm-nominal-diameter bolt the required minimum bolt tension is 210 kN. Assuming the coefficient of friction to be 0.40, determine the required couple that should be applied to the bolt and nut. The mean diameter of the thread is 22.6 mm, and the lead is 3 mm. Neglect friction between the nut and washer, and assume the bolt to be square-threaded.

SOLUTION

FBD block on incline:



$$\theta = \tan^{-1}\frac{3 \text{ mm}}{(22.6 \text{ mm})\pi}$$

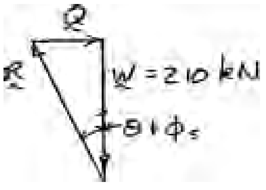
$$= 2.4195°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.40$$

$$\phi_s = 21.801°$$

$$Q = (210 \text{ kN})\tan(21.801° + 2.4195°)$$

$$Q = 94.468 \text{ kN}$$



$$\text{Torque} = \frac{d}{2}Q$$

$$= \frac{22.6 \text{ mm}}{2}(94.468 \text{ kN})$$

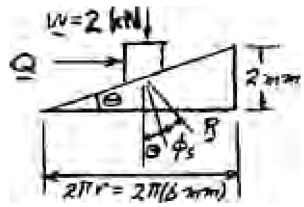
$$= 1067.49 \text{ N}\cdot\text{m}$$

Torque $= 1068 \text{ N}\cdot\text{m}$ ◀

PROBLEM 8.70

The ends of two fixed rods A and B are each made in the form of a single-threaded screw of mean radius 6 mm and pitch 2 mm. Rod A has a right-handed thread and rod B has a left-handed thread. The coefficient of static friction between the rods and the threaded sleeve is 0.12. Determine the magnitude of the couple that must be applied to the sleeve in order to draw the rods closer together.

SOLUTION



To draw rods together:

<u>Screw at A</u>

$$\tan\theta = \frac{2\text{ mm}}{2\pi(6\text{ mm})}$$

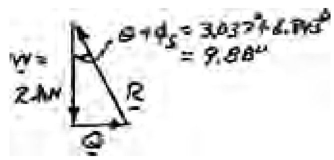
$$\theta = 3.037°$$

$$\phi_s = \tan^{-1} 0.12$$

$$= 6.843°$$

$$Q = (2\text{ kN})\tan 9.88°$$

$$= 348.3\text{ N}$$



$$\text{Torque at } A = Qr$$

$$= (348.3\text{ N})(6\text{ mm})$$

$$= 2.09\text{ N}\cdot\text{m}$$

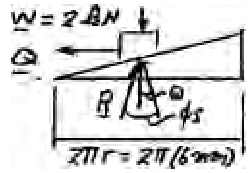
Same torque required at B Total torque $= 4.18\text{ N}\cdot\text{m}$ ◀

PROBLEM 8.71

Assuming that in Problem 8.70 a right-handed thread is used on *both* rods A and B, determine the magnitude of the couple that must be applied to the sleeve in order to rotate it.

PROBLEM 8.70 The ends of two fixed rods A and B are each made in the form of a single-threaded screw of mean radius 6 mm and pitch 2 mm. Rod A has a right-handed thread and rod B has a left-handed thread. The coefficient of static friction between the rods and the threaded sleeve is 0.12. Determine the magnitude of the couple that must be applied to the sleeve in order to draw the rods closer together.

SOLUTION



From the solution to Problem 8.70,

$$\text{Torque at } A = 2.09\ \text{N}\cdot\text{m}$$

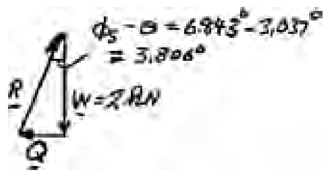
Screw at B: Loosening

$$\theta = 3.037^\circ$$

$$\phi_s = 6.843^\circ$$

$$Q = (2\ \text{kN})\tan 3.806^\circ$$

$$= 133.1\ \text{N}$$



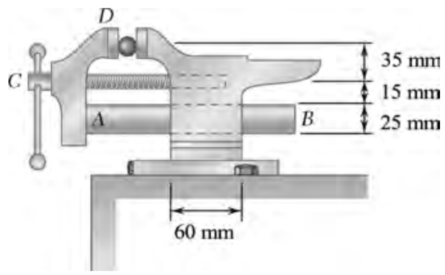
$$\text{Torque at } B = Qr$$

$$= (133.1\ \text{N})(6\ \text{mm})$$

$$= 0.798\ \text{N}\cdot\text{m}$$

$$\text{Total torque } = 2.09\ \text{N}\cdot\text{m} + 0.798\ \text{N}\cdot\text{m}$$

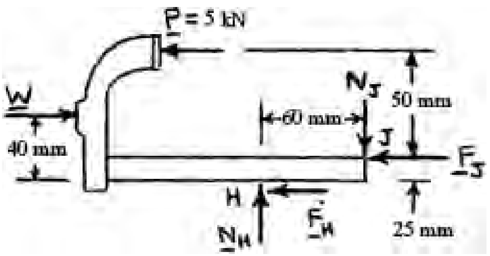
Total torque $= 2.89\ \text{N}\cdot\text{m}$ ◀



PROBLEM 8.72

In the machinist's vise shown, the movable jaw D is rigidly attached to the tongue AB that fits loosely into the fixed body of the vise. The screw is single-threaded into the fixed base and has a mean diameter of 15 mm and a pitch of 5 mm. The coefficient of static friction is 0.25 between the threads and also between the tongue and the body. Neglecting bearing friction between the screw and the mov able head, determine the couple that must be applied to the handle in order to produce a clamping force of 5 kN.

SOLUTION



Free body: Jaw D and tongue AB

P is due to elastic forces in clamped object.

W is force exerted by screw.

$$+\uparrow \Sigma F_y = 0: \quad N_H - N_J = 0 \quad N_J = N_H = N$$

For final tightening,

$$F_H = F_J = \mu_s N = 0.25\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad W - P - 2(0.25\text{ N}) = 0$$

$$N = 2(W - P) \tag{1}$$

$$+\circlearrowleft \Sigma M_H = 0: \quad P(75) - W(40) - N(60) + (0.25\text{ N})(25) = 0$$

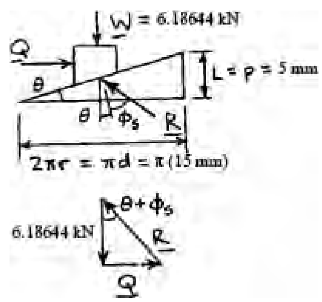
$$75P - 40W - 53.75\text{ N} = 0 \tag{2}$$

Substitute Eq. (1) into Eq. (2): $\quad 75P - 40W - 53.75[2(W - P)] = 0$

$$147.5W = 182.5P \Rightarrow W = \frac{182.5 \times 5}{147.5}\text{ kN}$$

$$W = 6.18644\text{ kN}$$

Block-and-incline analysis of screw:



$$\tan\phi_s = \mu_s = 0.25$$

$$\phi_s = 14.0362°$$

$$\tan\theta = \frac{5\text{ mm}}{\pi(15\text{ mm})}$$

$$\theta = 6.0566°$$

$$\theta + \phi_s = 20.093°$$

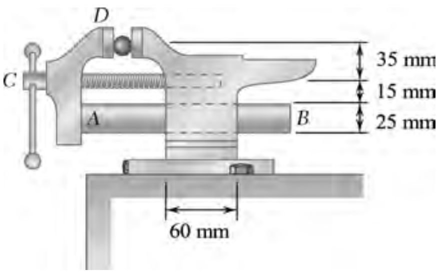
$$Q = (6.18644\text{ kN})\tan 20.093°$$

$$= 2.2631\text{ kN}$$

$$T = Qr = (2263.1\text{ N})\left(\frac{15 \times 10^{-3}\text{ m}}{2}\right)$$

$$T = 16.97\text{ N}\cdot\text{m} \blacktriangleleft$$

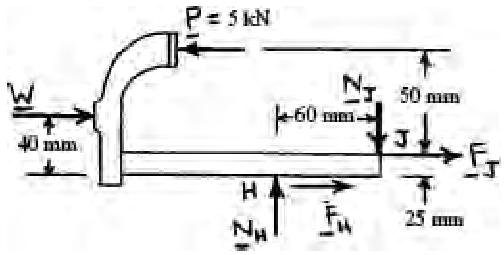
PROBLEM 8.73



In Problem 8.72, a clamping force of 5 kN was obtained by tightening the vise. Determine the couple that must be applied to the screw to loosen the vise.

PROBLEM 8.72 In the machinist's vise shown, the movable jaw *D* is rigidly attached to the tongue *AB* that fits loosely into the fixed body of the vise. The screw is single-threaded into the fixed base and has a mean diameter of 15 mm and a pitch of 5 mm. The coefficient of static friction is 0.25 between the threads and also between the tongue and the body. Neglecting bearing friction between the screw and the movable head, determine the couple that must be applied to the handle in order to produce a clamping force of 5 kN.

SOLUTION



<u>Free body: Jaw D and tongue *AB*</u>

P is due to elastic forces in clamped object.

W is force exerted by screw.

$$+\uparrow \Sigma F_y = 0: \quad N_H - N_J = 0 \quad N_J = N_H = N$$

As vise is just about to loosen, $F_H = F_J = \mu_s N = 0.25\text{ N}$

$$\xrightarrow{+} \Sigma F_x = 0: \quad W - P + 2(0.25\text{ N}) = 0$$

$$N = 2(P - W) \tag{1}$$

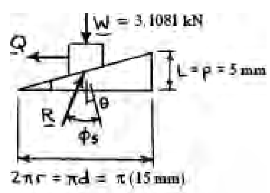
$$\circlearrowleft + \Sigma M_H = 0: \quad P(75) - W(40) - N(60) - (0.25\text{ N})(25) = 0$$

$$75P - 40W - 66.25\text{ N} = 0 \tag{2}$$

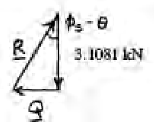
Substitute Eq. (1) into Eq. (2): $75P - 40W - 66.25[2(P - W)] = 0$

$$92.5W = 57.5P = 57.5(5\,\text{kN})$$

$$W = 3.1081\,\text{kN}$$



<u>Block-and-incline analysis of screw</u>:



$$\tan\phi_s = \mu_s = 0.25 \qquad \phi_s = 14.0362°$$

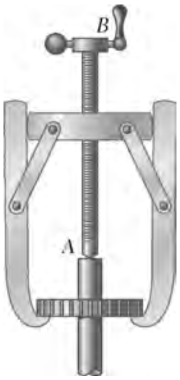
$$\tan\theta = \frac{5\,\text{mm}}{\pi(15\,\text{mm})} \qquad \theta = 6.0566°$$

$$\phi_s - \theta = 7.9796°$$

$$Q = (3.1081\,\text{kN})\tan 7.9796° = 0.43569\text{ kN} = 435.69\text{ N}$$

$$M = Qr = (435.69\text{ N})\left(\frac{15\times 10^{-3}\,\text{m}}{2}\right)$$

$M = 3.27\text{ N·m}$ ◀

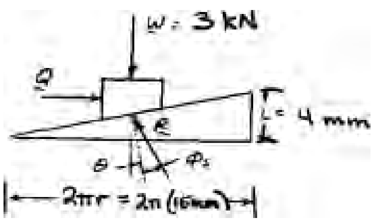


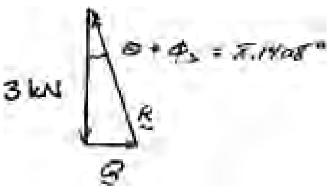
PROBLEM 8.74

In the gear-pulling assembly shown the square-threaded screw AB has a mean radius of 15 mm and a lead of 4 mm. Knowing that the coefficient of static friction is 0.10, determine the couple that must be applied to the screw in order to produce a force of 3 kN on the gear. Neglect friction at end A of the screw.

SOLUTION

Block / Incline





$$\theta = \tan^{-1}\frac{6.35\text{ mm}}{47.625\pi\text{ mm}}$$ TQ1

$$= 2.4302°$$

$$\phi_s = \tan^{-1}\mu_s$$

$$= \tan^{-1}(0.10)$$

$$= 5.7106°$$

$$Q = (3000\text{ N})\tan(8.1408°)$$

$$= 429.14\text{ N}$$

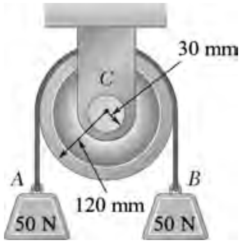
$$\text{Couple} = rQ$$

$$= (0.015\text{ m})(429.14\text{ N})$$

$$= 6.4371\text{ N}\cdot\text{m}$$

$M = 6.44\text{ N}\cdot\text{m}$ ◀

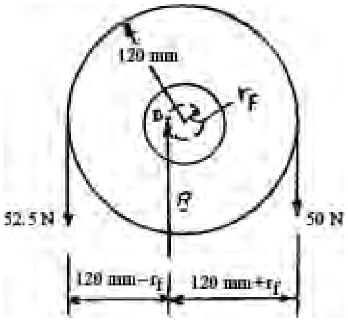
TQ1: Please check this value



PROBLEM 8.75

A 120 mm-radius pulley of weight 25 N is attached to a 30 mm-radius shaft that fits loosely in a fixed bearing. It is observed that the pulley will just start rotating if a 2.5-N weight is added to block *A*. Determine the coefficient of static friction between the shaft and the bearing.

SOLUTION



$$+\circlearrowleft\ \Sigma M_D = 0: \quad (52.5\text{ N})(120\text{ mm} - r_f) - (50\text{ N})(120\text{ mm} + r_f) = 0$$

$$(2.5\text{ N})(120\text{ mm}) = (102.5\text{ N})r_f$$

$$r_f = 2.9268\text{ mm}$$

$$r_f = r\sin\phi_s$$

$$\sin\varphi_s = \frac{2.9268\text{ mm}}{30\text{ mm}}$$

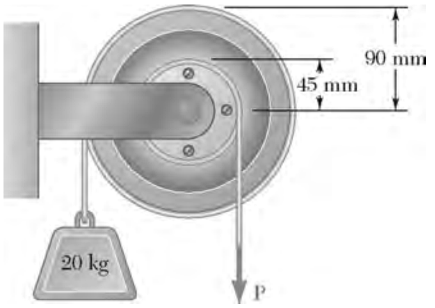
$$= 0.097561$$

$$\phi_s = 5.5987°$$

$$\mu_s = \tan\phi_s$$

$$= \tan 5.5987°$$

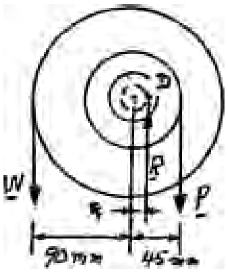
$$\mu_s = 0.0980 \blacktriangleleft$$



PROBLEM 8.76

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to start raising the load.

SOLUTION



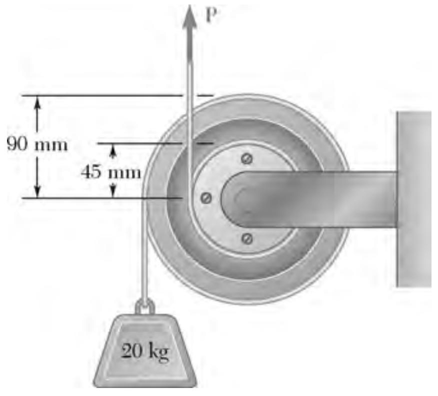
$$+\circlearrowleft\ \Sigma M_D = 0: \quad P(45 - r_f) - W(90 + r_f) = 0$$

$$P = W\frac{90 + r_f}{45 - r_f}$$

$$= (196.2 \text{ N})\frac{90 \text{ mm} + 4 \text{ mm}}{45 \text{ mm} - 4 \text{ mm}}$$

$$P = 449.82 \text{ N}$$

P = 450 N ↓ ◀

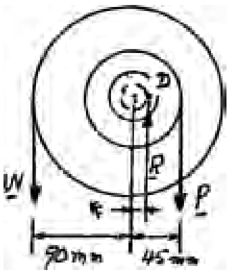


PROBLEM 8.77

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to start raising the load.

SOLUTION

Find **P** required to start raising load

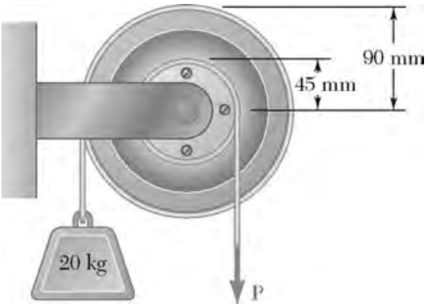


$$+\circlearrowleft \Sigma M_D = 0: \quad P(45 - r_f) - W(90 - r_f) = 0$$

$$P = W\frac{90 - r_f}{45 - r_f}$$

$$= (196.2\text{ N})\frac{90\text{ mm} - 4\text{ mm}}{45\text{ mm} - 4\text{ mm}}$$

$$P = 411.54\text{ N} \qquad \mathbf{P} = 412\text{ N}\uparrow \blacktriangleleft$$

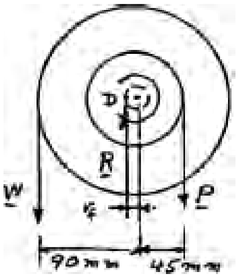


PROBLEM 8.78

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the smallest force **P** required to maintain equilibrium.

SOLUTION

<u>Find smallest **P** to maintain equilibrium</u>



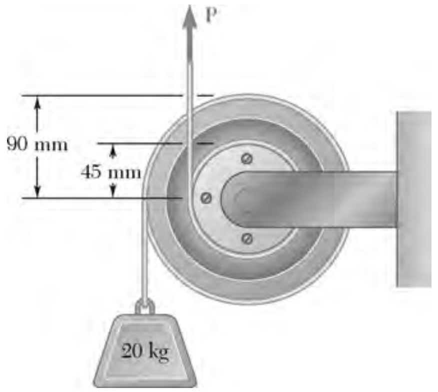
$$+\circlearrowleft \Sigma M_D = 0: \quad P(45 + r_f) - W(90 - r_f) = 0$$

$$P = W\frac{90 - r_f}{45 + r_f}$$

$$= (196.2\text{ N})\frac{90\text{ mm} - 4\text{ mm}}{45\text{ mm} + 4\text{ mm}}$$

$$P = 344.35\text{ N}$$

$\mathbf{P} = 344\text{ N}\downarrow$ ◀

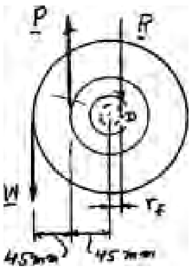


PROBLEM 8.79

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the smallest force **P** required to maintain equilibrium.

SOLUTION

Find smallest **P** to maintain equilibrium



$$+\circlearrowleft \Sigma M_D = 0: \quad P(45 + r_f) - W(90 + r_f) = 0$$

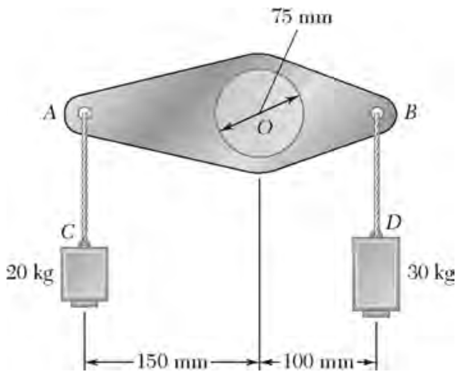
$$P = W\frac{90 + r_f}{45 + r_f}$$

$$= (196.2\text{ N})\frac{90\text{ mm} + 4\text{ mm}}{45\text{ mm} + 4\text{ mm}}$$

$$P = 376.38\text{ N}$$

$\mathbf{P} = 376\text{ N}\uparrow$ ◀

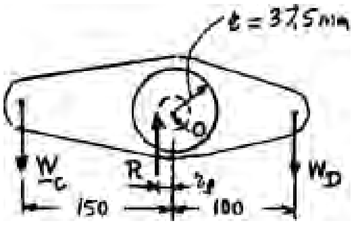
PROBLEM 8.80



A lever of negligible weight is loosely fitted onto a 75-mm-diameter fixed shaft. It is observed that the lever will just start rotating if a 3-kg mass is added at *C*. Determine the coefficient of static friction between the shaft and the lever.

SOLUTION

$$+\circlearrowleft \Sigma M_O = 0: \quad W_C(150) - W_D(100) - Rr_f = 0$$



But $$W_C = (23\text{ kg})(9.81\text{ m/s}^2)$$

$$W_D = (30\text{ kg})(9.81\text{ m/s}^2)$$

$$R = W_C + W_D = (53\text{ kg})(9.81)$$

Thus, after dividing by 9.81,

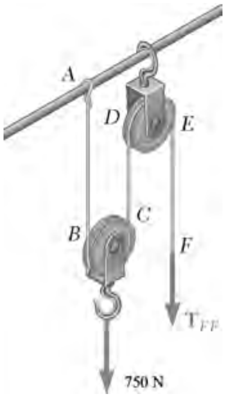
$$23(150) - 30(100) - 53\,r_f = 0$$

$$r_f = 8.49\text{ mm}$$

But $$\mu_s \approx \frac{r_f}{r} = \frac{8.49\text{ mm}}{37.5\text{ mm}}$$

$$\mu_s = 0.226 \blacktriangleleft$$

PROBLEM 8.81

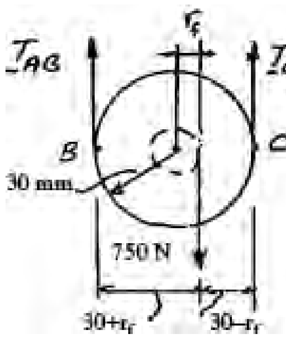


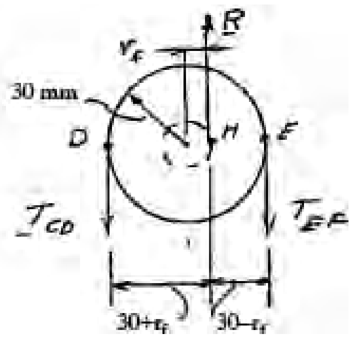
The block and tackle shown are used to raise a 750-N load. Each of the 60-mm-diameter pulleys rotates on a 10-mm-diameter axle. Knowing that the coefficient of static friction is 0.20, determine the tension in each portion of the rope as the load is slowly raised.

SOLUTION

For each pulley: Axle diameter $= 10$ mm

$$r_f = r\sin\phi_s \approx \mu_s r = 0.20\left(\frac{10\text{ mm}}{2}\right) = 1\text{ mm}$$





Pulley *BC*:

$$+\circlearrowleft \Sigma M_B = 0: \quad T_{CD}(60\text{ mm}) - (750\text{ N})(30\text{ mm} + r_f) = 0$$

$$T_{CD} = \frac{1}{60}(750\text{ N})(30\text{ mm} + 1\text{ mm}) = 387.5\text{ N} \qquad T_{CD} = 388\text{ N} \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad T_{AB} + 387.5\text{ N} - 750\text{ N} = 0 \Rightarrow T_{AB} = 362.5\text{ N} \qquad T_{AB} = 363\text{ N} \blacktriangleleft$$

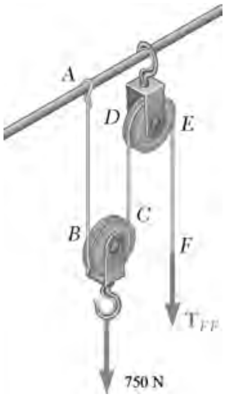
Pulley *DE*:

$$+\circlearrowleft \Sigma M_B = 0: \quad T_{CD}(30 + r_f) - T_{EF}(30 - r_f) = 0$$

$$T_{EF} = T_{CD}\frac{30 + r_f}{30 - r_f}$$

$$= (387.5\text{ N})\frac{30\text{ mm} + 1\text{ mm}}{30\text{ mm} - 1\text{ mm}} = 414.22\text{ N} \qquad T_{EF} = 414\text{ N} \blacktriangleleft$$

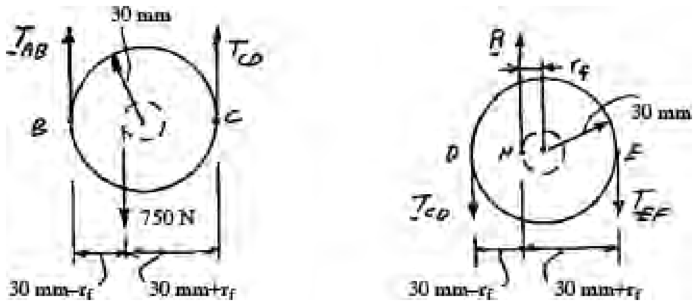
PROBLEM 8.82



The block and tackle shown are used to lower a 750-N load. Each of the 60-mm-diameter pulleys rotates on a 10 mm-diameter axle. Knowing that the coefficient of static friction is 0.20, determine the tension in each portion of the rope as the load is slowly lowered.

SOLUTION

For each pulley: $r_f = r\mu_s = \left(\dfrac{10\text{ mm}}{2}\right)0.2 = 1\text{ mm}$



Pulley *BC*: $\circlearrowleft + \Sigma M_B = 0: \quad T_{CD}(60\text{ mm}) - (750\text{ N})(30\text{ mm} - r_f) = 0$

$$T_{CD} = \frac{(750\text{ N})(30\text{ mm} - 1\text{ mm})}{60\text{ mm}} = 362.5\text{ N} \qquad T_{CD} = 363\text{ N} \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad T_{AB} + 362.5\text{ N} - 750\text{ N} = 0 \Rightarrow T_{AB} = 387.5\text{ N} \qquad T_{AB} = 388\text{ N} \blacktriangleleft$$

Pulley *DE*: $T_{CD}(30\text{ mm} - r_f) - T_{EF}(30\text{ mm} + r_f) = 0$

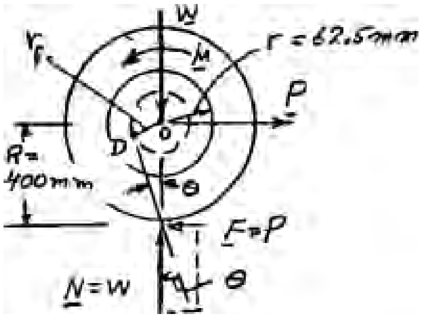
$$T_{EF} = T_{CD}\frac{30\text{ mm} - r_f}{30\text{ mm} + r_f}$$

$$= (362.5\text{ N})\frac{30\text{ mm} - 1\text{ mm}}{30\text{ mm} + 1\text{ mm}} = 339.11\text{ N} \qquad T_{EF} = 339\text{ N} \blacktriangleleft$$

PROBLEM 8.83

A loaded railroad car has a mass of 30 Mg and is supported by eight 800-mm-diameter wheels with 125-mm-diameter axles. Knowing that the coefficients of friction are $\mu_s = 0.020$ and $\mu_k = 0.015$, determine the horizontal force required (*a*) to start the car moving, (*b*) to keep the car moving at a constant speed. Neglect rolling resistance between the wheels and the track.

SOLUTION



$$r_f = \mu r; \quad R = 400 \text{ mm}$$

$$\sin\theta = \tan\theta = \frac{r_f}{R} = \frac{\mu r}{R}$$

$$P = W \tan\theta = W\frac{\mu r}{R}$$

$$P = W\mu\frac{62.5 \text{ mm}}{400 \text{ mm}}$$

$$= 0.15625 W\mu$$

For one wheel:

$$W = \frac{1}{8}(30 \text{ mg})(9.81 \text{ m/s}^2)$$

$$= \frac{1}{8}(294.3 \text{ kN})$$

For eight wheels of rail road car:

$$\Sigma P = 8(0.15625)\frac{1}{8}(294.3 \text{ kN})\mu$$

$$= (45.984\mu) \text{ kN}$$

(*a*) To start motion:

$$\mu_s = 0.020$$

$$\Sigma P = (45.984)(0.020)$$

$$= 0.9197 \text{ kN}$$

$\Sigma P = 920$ N ◀

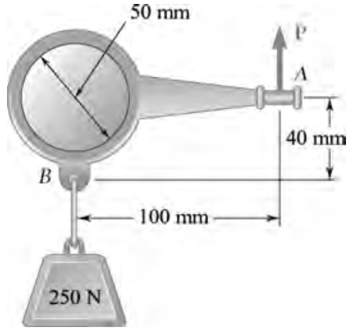
(*b*) To maintain motion:

$$\mu_k = 0.015$$

$$\Sigma P = (45.984)(0.015)$$

$$= 0.6897 \text{ kN}$$

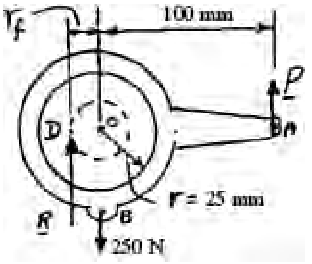
$\Sigma P = 690$ N ◀



PROBLEM 8.84

A lever *AB* of negligible weight is loosely fitted onto a 50-mm-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating counterclockwise.

SOLUTION

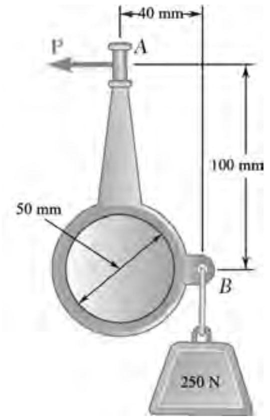


$$r_f = \mu_s r$$
$$= 0.15(25 \text{ mm})$$
$$= 3.75 \text{ mm}$$

$$\circlearrowleft + \Sigma M_D = 0: \quad P(100 \text{ mm} + r_f) - (250 \text{ N})r_f = 0$$

$$P = \frac{250(3.75)}{103.75}$$
$$= 9.0361 \text{ N}$$

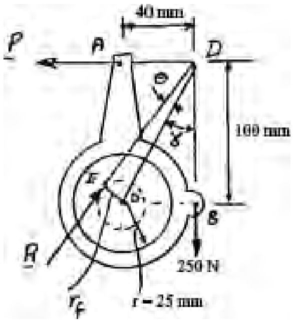
P = 9.04 N ↑ ◀



PROBLEM 8.85

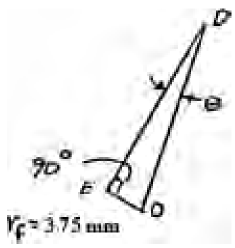
A lever *AB* of negligible weight is loosely fitted onto a 50 mm-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating counterclockwise.

SOLUTION



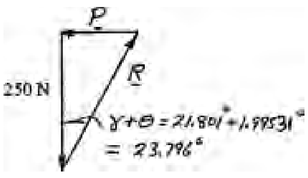
$$r_f = \mu_s r$$
$$= 0.15(25 \text{ mm})$$
$$r_f = 3.75 \text{ mm}$$
$$\tan\gamma = \frac{40 \text{ mm}}{100 \text{ mm}}$$
$$\gamma = 21.801°$$

In EOD:



$$OD = \sqrt{(40 \text{ mm})^2 + (100 \text{ mm})^2}$$
$$= 107.703 \text{ mm}$$
$$\sin\theta = \frac{OE}{OD} = \frac{r_f}{OD}$$
$$= \frac{3.75 \text{ mm}}{107.703 \text{ mm}}$$
$$\theta = 1.99532°$$

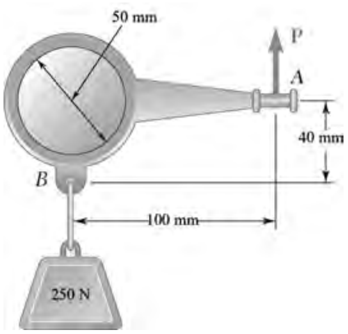
Force triangle



$$P = (250 \text{ N})\tan(\gamma + \theta)$$
$$= (250 \text{ N})\tan 23.796°$$
$$= 110.243 \text{ N}$$

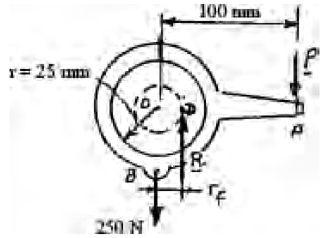
$\mathbf{P} = 110.2 \text{ N}$ ◀

PROBLEM 8.86



A lever *AB* of negligible weight is loosely fitted onto a 50-mm-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating clockwise.

SOLUTION

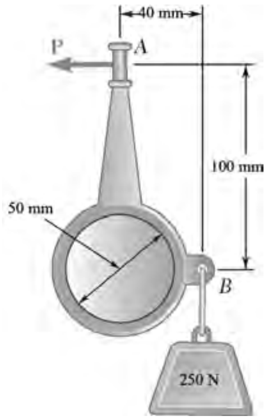


$$r_f = \mu_s r$$
$$= 0.15(25 \text{ mm})$$
$$r_f = 3.75 \text{ mm}$$

$$+\circlearrowright \Sigma M_D = 0: \quad P(100 \text{ mm} - r_f) - (250 \text{ N})r_f = 0$$

$$P = \frac{250(3.75)}{100 - 3.75}$$
$$= 9.7403 \text{ N}$$

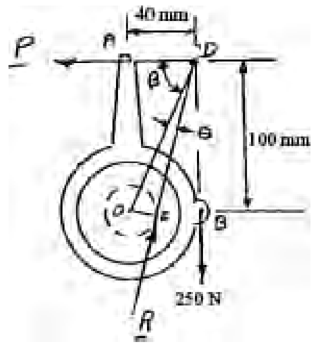
$\mathbf{P} = 9.74 \text{ N} \downarrow$ ◀



PROBLEM 8.87

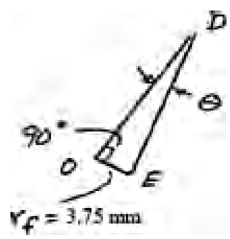
A lever AB of negligible weight is loosely fitted onto a 50-mm-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating clockwise.

SOLUTION



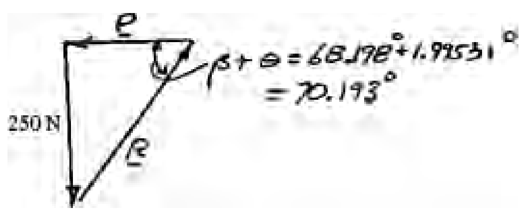
$$r_f = \mu_s r$$
$$= 0.15(25\text{ mm})$$
$$= 3.75\text{ mm}$$
$$\tan\beta = \frac{100\text{ mm}}{40\text{ mm}}$$
$$\beta = 68.198°$$

In ΔEOD:



$$OD = \sqrt{(40)^2 + (100)^2}$$
$$OD = 107.703\text{ mm}$$
$$\sin\theta = \frac{OE}{OD} = \frac{3.75\text{ mm}}{107.703\text{ mm}}$$
$$\theta = 1.99532°$$

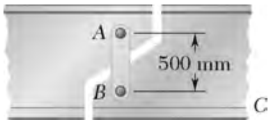
Force triangle:



$$P = \frac{250}{\tan(\beta + \theta)} = \frac{250\text{ N}}{\tan 70.193°} = 90.04\text{ N}$$

$\mathbf{P} = 90.0\text{ N} \leftarrow$ ◀

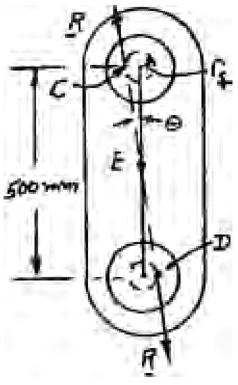
PROBLEM 8.88



The link arrangement shown is frequently used in highway bridge construction to allow for expansion due to changes in temperature. At each of the 60-mm-diameter pins *A* and *B* the coefficient of static friction is 0.20. Knowing that the vertical component of the force exerted by *BC* on the link is 200 kN, determine (*a*) the horizontal force that should be exerted on beam *BC* to just move the link, (*b*) the angle that the resulting force exerted by beam *BC* on the link will form with the vertical.

SOLUTION

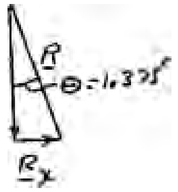
Bearing:



$$r = 30 \text{ mm}$$
$$r_f = \mu_s r$$
$$= 0.20(30 \text{ mm})$$
$$= 6 \text{ mm}$$

Resultant forces **R** must be tangent to friction circles at Points *C* and *D*.

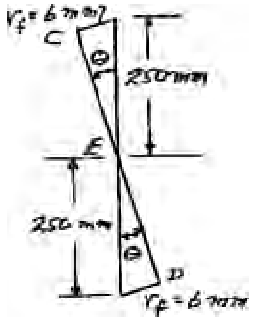
(*a*)



$$R_y = \text{Vertical component} = 200 \text{ kN}$$
$$R_x = R_y \tan\theta$$
$$= (200 \text{ kN}) \tan 1.375°$$
$$= 4.80 \text{ kN}$$

Horizontal force = 4.80 kN ◀

(*b*)



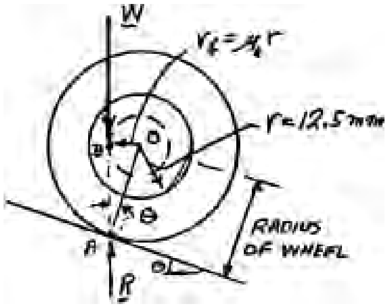
$$\sin\theta = \frac{6 \text{ mm}}{250 \text{ mm}}$$
$$\sin\theta = 0.024$$

$\theta = 1.375°$ ◀

PROBLEM 8.89

A scooter is to be designed to roll down a 2 percent slope at a constant speed. Assuming that the coefficient of kinetic friction between the 25-mm-diameter axles and the bearings is 0.10, determine the required diameter of the wheels. Neglect the rolling resistance between the wheels and the ground.

SOLUTION



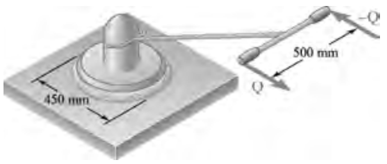
$$\tan\theta = \frac{2}{100} = 0.02$$

Since a scooter rolls at constant speed, each wheel is in equilibrium. Thus, **W** and **R** must have a common line of action tangent to the friction circle.

$$r_f = \mu_k r = (0.10)(12.5 \text{ mm})$$
$$= 1.25 \text{ mm}$$
$$OA = \frac{OB}{\tan\theta} = \frac{r_f}{\tan\theta} = \frac{1.25 \text{ mm}}{0.02}$$
$$= 62.5 \text{ mm}$$

Diameter of wheel $= 2(OA) = 125.0$ mm ◄

PROBLEM 8.90



A 250-N electric floor polisher is operated on a surface for which the coefficient of kinetic friction is 0.25. Assuming that the normal force per unit area between the disk and the floor is uniformly distributed, determine the magnitude Q of the horizontal forces required to prevent motion of the machine.

SOLUTION

See Figure 8.12 and Eq. (8.9).

Using:

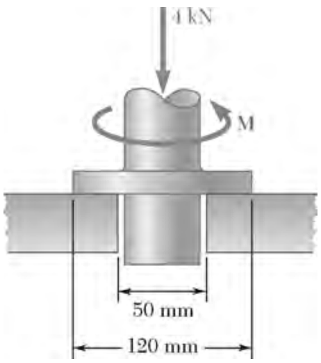
$$R = 225 \text{ mm} = 0.225 \text{ m}$$
$$P = 250 \text{ N}$$

and

$$\mu_k = 0.25$$
$$M = \frac{2}{3}\mu_k PR = \frac{2}{3}(0.25)(250 \text{ N})(0.225 \text{ m})$$
$$= 9.375 \text{ N} \cdot \text{m}$$

$\Sigma M_y = 0$ yields:
$$M = Q(0.5 \text{ m})$$
$$9.375 \text{ N} \cdot \text{m} = Q(0.5 \text{ m})$$

$Q = 18.75$ N ◀



PROBLEM 8.91

Knowing that a couple of magnitude $30\text{ N}\cdot\text{m}$ is required to start the vertical shaft rotating, determine the coefficient of static friction between the annular surfaces of contact.

SOLUTION

For annular contact regions, use Equation 8.8 with impending slipping:

$$M = \frac{2}{3}\mu_s N \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

So,

$$30\text{ N}\cdot\text{m} = \frac{2}{3}\mu_s(4000\text{ N})\frac{(0.06\text{ m})^3 - (0.025\text{ m})^3}{(0.06\text{ m})^2 - (0.025\text{ m})^2}$$

$$\mu_s = 0.1670 \blacktriangleleft$$

PROBLEM 8.92*

The frictional resistance of a thrust bearing decreases as the shaft and bearing surfaces wear out. It is generally assumed that the wear is directly proportional to the distance traveled by any given point of the shaft and thus to the distance r from the point to the axis of the shaft. Assuming, then, that the normal force per unit area is inversely proportional to r, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out end bearing (with contact over the full circular area) is equal to 75 percent of the value given by Eq. (8.9) for a new bearing.

SOLUTION

Using Figure 8.12, we assume

$$\Delta N = \frac{k}{r}\Delta A: \quad \Delta A = r\Delta\theta\,\Delta r$$

$$\Delta N = \frac{k}{r} r\Delta\theta\Delta r = k\Delta\theta\Delta r$$

We write

$$P = \Sigma\Delta N \quad \text{or} \quad P = \int dN$$

$$P = \int_0^{2\pi}\int_0^R k\Delta\theta\Delta r = 2\pi Rk; \quad k = \frac{P}{2\pi R}$$

$$\Delta N = \frac{P\Delta\theta\Delta r}{2\pi R}$$

$$\Delta M = r\Delta F = r\mu_k\Delta N = r\mu_k\frac{P\Delta\theta\Delta r}{2\pi R}$$

$$M = \int_0^{2\pi}\int_0^R \frac{\mu_k P}{2\pi R} r dr d\theta = \frac{2\pi\mu_k P}{2\pi R}\cdot\frac{R^2}{2} = \frac{1}{2}\mu_k PR$$

From Eq. (8.9) for a new bearing,

$$M_{\text{New}} = \frac{2}{3}\mu_k PR$$

Thus,

$$\frac{M}{M_{\text{New}}} = \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{3}{4}$$

$M = 0.75M_{\text{New}}$ ◀

PROBLEM 8.93*

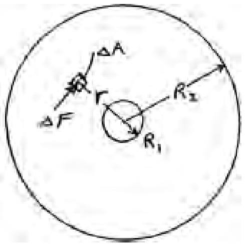
Assuming that bearings wear out as indicated in Problem 8.92, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out collar bearing is

$$M = \frac{1}{2}\mu_k P(R_1 + R_2)$$

where P = magnitude of the total axial force

R_1, R_2 = inner and outer radii of collar

SOLUTION



Let normal force on ΔA be ΔN, and $\dfrac{\Delta N}{\Delta A} = \dfrac{k}{r}$.

As in the text, $\Delta F = \mu \Delta N \quad \Delta M = r\Delta F$

The total normal force P is

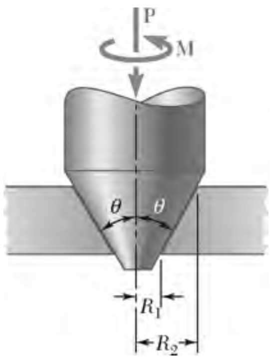
$$P = \lim_{\Delta A \to 0} \Sigma \Delta N = \int_0^{2\pi}\left(\int_{R_1}^{R_2} \frac{k}{r} r dr\right) d\theta$$

$$P = 2\pi \int_{R_1}^{R_2} k dr = 2\pi k(R_2 - R_1) \quad \text{or} \quad k = \frac{P}{2\pi(R_2 - R_1)}$$

Total couple: $$M_{\text{worn}} = \lim_{\Delta A \to 0} \Sigma \Delta M = \int_0^{2\pi}\left(\int_{R_1}^{R_2} r\mu \frac{k}{r} r dr\right) d\theta$$

$$M_{\text{worn}} = 2\pi\mu k \int_{R_1}^{R_2} (r dr) = \pi\mu k\left(R_2^2 - R_1^2\right) = \frac{\pi\mu P\left(R_2^2 - R_1^2\right)}{2\pi(R_2 - R_1)}$$

$$M_{\text{worn}} = \frac{1}{2}\mu P(R_2 + R_1) \blacktriangleleft$$



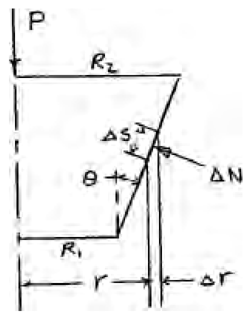
PROBLEM 8.94*

Assuming that the pressure between the surfaces of contact is uniform, show that the magnitude M of the couple required to overcome frictional resistance for the conical bearing shown is

$$M = \frac{2}{3}\frac{\mu_k P}{\sin\theta}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

SOLUTION

Let normal force on ΔA be ΔN and $\dfrac{\Delta N}{\Delta A} = k.$



So $\qquad \Delta N = k\Delta A \qquad \Delta A = r\Delta s\Delta\phi \qquad \Delta s = \dfrac{\Delta r}{\sin\theta}$

where ϕ is the azimuthal angle around the symmetry axis of rotation.

$$\Delta F_y = \Delta N \sin\theta = kr\Delta r\Delta\phi$$

Total vertical force: $\qquad P = \lim_{\Delta A \to 0} \Sigma\Delta F_y$

$$P = \int_0^{2\pi}\left(\int_{R_1}^{R_2} krdr\right)d\phi = 2\pi k\int_{R_1}^{R_2} rdr$$

$$P = \pi k\left(R_2^2 - R_1^2\right) \quad \text{or} \quad k = \frac{P}{\pi\left(R_2^2 - R_1^2\right)}$$

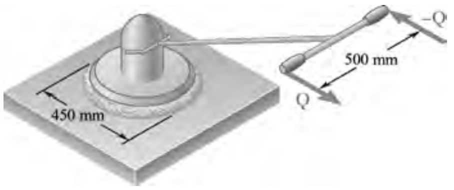
Friction force: $\qquad \Delta F = \mu\Delta N = \mu k\Delta A$

Moment: $\qquad \Delta M = r\Delta F = r\mu kr\dfrac{\Delta r}{\sin\theta}\Delta\phi$

Total couple: $\qquad M = \lim_{\Delta A \to 0}\Sigma\Delta M = \displaystyle\int_0^{2\pi}\left(\int_{R_1}^{R_2}\frac{\mu k}{\sin\theta}r^2dr\right)d\phi$

$$M = 2\pi\frac{\mu k}{\sin\theta}\int_{R_1}^{R_2} r^2dr = \frac{2}{3}\frac{\pi\mu}{\sin\theta}\frac{P}{\pi\left(R_2^2 - R_3^2\right)}\left(R_2^3 - R_3^3\right)$$

$$M = \frac{2}{3}\frac{\mu P}{\sin\theta}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \blacktriangleleft$$



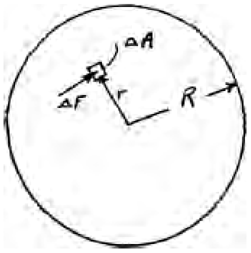
PROBLEM 8.95

Solve Problem 8.90 assuming that the normal force per unit area between the disk and the floor varies linearly from a maximum at the center to zero at the circumference of the disk.

PROBLEM 8.90 A 250-N electric floor polisher is operated on a surface for which the coefficient of kinetic friction is 0.25. Assuming that the normal force per unit area between the disk and the floor is uniformly distributed, determine the magnitude Q of the horizontal forces required to prevent motion of the machine.

SOLUTION

Let normal force on ΔA be ΔN and $\dfrac{\Delta N}{\Delta A} = k\left(1-\dfrac{r}{R}\right)$.



$$\Delta F = \mu \Delta N = \mu k\left(1-\frac{r}{R}\right)\Delta A = \mu k\left(1-\frac{r}{R}\right) r\Delta r\Delta\theta$$

$$P = \lim_{\Delta A\to 0} \Sigma\Delta N = \int_0^{2\pi}\left[\int_0^R k\left(1-\frac{r}{R}\right) r dr\right] d\theta$$

$$P = 2\pi k\int_0^R\left(1-\frac{r}{R}\right) r dr = 2\pi k\left(\frac{R^2}{2}-\frac{R^3}{3R}\right)$$

$$P = \frac{1}{3}\pi k R^2 \quad \text{or} \quad k = \frac{3P}{\pi R^2}$$

$$M = \lim_{\Delta A\to 0} \Sigma r\Delta F = \int_0^{2\pi}\left[\int_0^R r\mu k\left(1-\frac{r}{R}\right) r dr\right] d\theta$$

$$= 2\pi\mu k\int_0^R\left(r^2-\frac{r^3}{R}\right) dr$$

$$= 2\pi\mu k\left(\frac{R^3}{3}-\frac{R^4}{4R}\right) = \frac{1}{6}\pi\mu k R^3$$

$$= \frac{\pi\mu}{6}\frac{3P}{\pi R^2}R^3 = \frac{1}{2}\mu P R$$

where $\mu = \mu_k = 0.25 \quad R = 0.225 \text{ m}$

PROBLEM 8.95 (Continued)

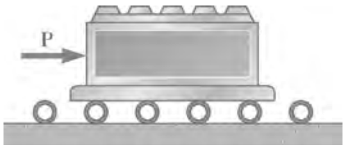
$$P = W = 250 \text{ N}$$

Then
$$M = \frac{1}{2}(0.25)(250 \text{ N})(0.225 \text{ m})$$
$$= 7.03125 \text{ N} \cdot \text{m}$$

Finally
$$Q = \frac{M}{d} = \frac{7.03125 \text{ N} \cdot \text{m}}{0.5 \text{ m}} = 14.0625 \text{ N}$$

$Q = 14.06 \text{ N}$ ◀

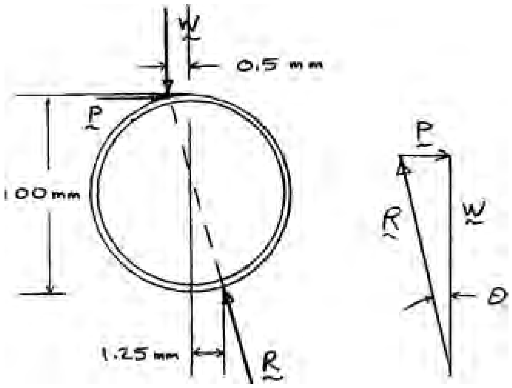
PROBLEM 8.96



A 900-kg machine base is rolled along a concrete floor using a series of steel pipes with outside diameters of 100 mm. Knowing that the coefficient of rolling resistance is 0.5 mm between the pipes and the base and 1.25 mm between the pipes and the concrete floor, determine the magnitude of the force **P** required to slowly move the base along the floor.

SOLUTION

FBD pipe:



$$W = mg$$
$$= (900 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 8829.0 \text{ N}$$
$$\theta = \sin^{-1} \frac{0.5 \text{ mm} + 1.25 \text{ mm}}{100 \text{ mm}}$$
$$= 1.00273°$$

$P = W \tan\theta$ for each pipe, so also for total

$$P = (8829.0 \text{ N}) \tan(1.00273°)$$

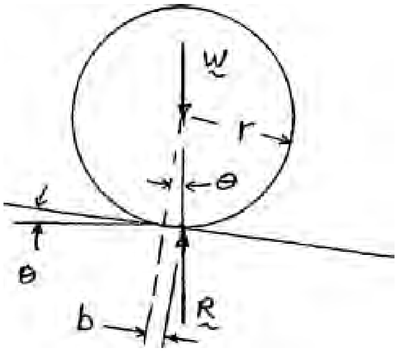
$\mathbf{P} = 154.4 \text{ N}$ ◄

PROBLEM 8.97

Knowing that a 150-mm-diameter disk rolls at a constant velocity down a 2 percent incline, determine the coefficient of rolling resistance between the disk and the incline.

SOLUTION

FBD disk:



$$\tan\theta = \text{slope} = 0.02$$

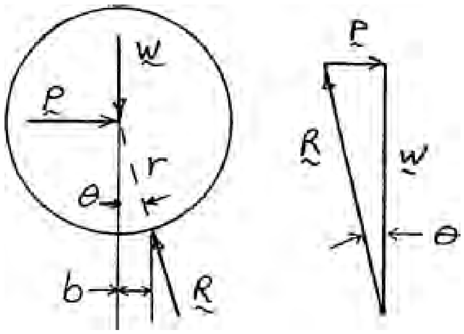
$$b = r\tan\theta = (150 \text{ mm})(0.02) \qquad b = 3.00 \text{ mm} ◀$$

PROBLEM 8.98

Determine the horizontal force required to move a 10-kN automobile with 600-mm-diameter tires along a horizontal road at a constant speed. Neglect all forms of friction except rolling resistance, and assume the coefficient of rolling resistance to be 1.5 mm.

SOLUTION

FBD wheel:



$$r = 300 \text{ mm}$$

$$b = 1.5 \text{ mm}$$

$$\theta = \sin^{-1}\frac{b}{r}$$

$P = W\tan\theta = W\tan\left(\sin^{-1}\dfrac{b}{r}\right)$ for each wheel, so for total

$$P = 10{,}000\text{ N}\tan\left(\sin^{-1}\frac{1.5}{300}\right)$$

$P = 50.0$ N ◀

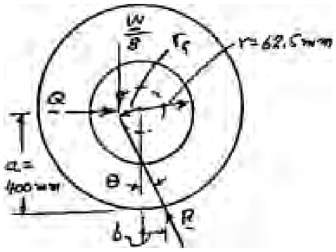
PROBLEM 8.99

Solve Problem 8.83 including the effect of a coefficient of rolling resistance of 0.5 mm.

PROBLEM 8.83 A loaded railroad car has a mass of 30 Mg and is supported by eight 800-mm-diameter wheels with 125-mm-diameter axles. Knowing that the coefficients of friction are $\mu_s = 0.020$ and $\mu_k = 0.015$, determine the horizontal force required (*a*) to start the car moving, (*b*) to keep the car moving at a constant speed. Neglect rolling resistance between the wheels and the track.

SOLUTION

For one wheel:



$$r_f = \mu r$$

$$\tan\theta \approx \sin\theta \approx \frac{r_f + b}{a}$$

$$\tan\theta = \frac{\mu r + b}{a}$$

$$Q = \frac{W}{8}\tan\theta = \frac{W}{8}\frac{\mu r + b}{a}$$

For eight wheels of car:

$$P = W\frac{\mu r + b}{a}$$

$$W = mg = (30 \text{ Mg})(9.81 \text{ m/s}^2) = 294.3 \text{ kN}$$

$$a = 400 \text{ mm}, \quad r = 62.5 \text{ mm}, \quad b = 0.5 \text{ mm}$$

(*a*) To start motion:

$$\mu = \mu_s = 0.02$$

$$P = (294.3 \text{ kN})\frac{(0.020)(62.5 \text{ mm}) + 0.5 \text{ mm}}{400 \text{ mm}}$$

$$P = 1.288 \text{ kN} \blacktriangleleft$$

(*b*) To maintain constant speed

$$\mu = \mu_k = 0.015$$

$$P = (294.3 \text{ kN})\frac{(0.015)(62.5 \text{ mm}) + 0.5 \text{ mm}}{400 \text{ mm}}$$

$$P = 1.058 \text{ kN} \blacktriangleleft$$

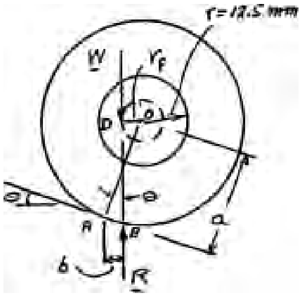
PROBLEM 8.100

Solve Problem 8.89 including the effect of a coefficient of rolling resistance of 1.75 mm.

PROBLEM 8.89 A scooter is to be designed to roll down a 2 percent slope at a constant speed. Assuming that the coefficient of kinetic friction between the 25-mm-diameter axles and the bearings is 0.10, determine the required diameter of the wheels. Neglect the rolling resistance between the wheels and the ground.

SOLUTION

Since the scooter rolls at a constant speed, each wheel is in equilibrium. Thus, **W** and **R** must have a common line of action tangent to the friction circle.



$$a = \text{Radius of wheel}$$

$$\tan\theta = \frac{2}{100} = 0.02$$

Since b and r_f are small compared to a,

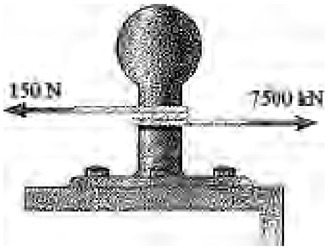
$$\tan\theta \approx \frac{r_f + b}{a} = \frac{\mu_k r + b}{a} = 0.02$$

Data: $\mu_k = 0.10, \quad b = 1.75 \text{ mm}, \quad r = 12.5 \text{ mm}$

$$\frac{(0.10)(12.5 \text{ mm}) + 1.75 \text{ mm}}{a} = 0.02$$

$$a = 150 \text{ mm}$$

Diameter $= 2a = 300$ mm ◀

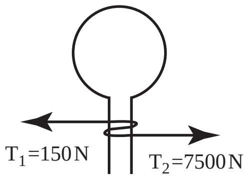


PROBLEM 8.101

A hawser thrown from a ship to a pier is wrapped two full turns around a bollard. The tension in the hawser is 7500-N; by exerting a force of 150- N on its free end, a dockworker can just keep the hawser from slipping. (*a*) Determine the cofficient of friction between the hawser and the bollard. (*b*) Determine the tension in the hawser that could be resisted by the 150-N force if the hawser were wrapped three full turns around the bollard.

SOLUTION

(*a*) **Coefficient of Friction.**



$$\ln \frac{T_2}{T_1} = \mu_s \beta$$

Since the hawser is wrapped two full turns around the bollard,

$$\beta = 2(2\pi \text{ rad}) = 12.57 \text{ rad}$$

$$T_1 = 150 \text{ N} \quad T_2 = 7500 \text{ N}$$

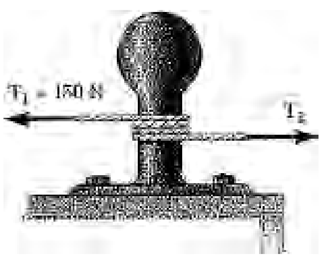
$$\mu_s \beta = \ln \frac{T_2}{T_1}$$

$$\mu_s (12.57 \text{ rad}) = \ln \frac{7500 \text{ N}}{150 \text{ N}} = \ln 50 = 3.91$$

$$\mu_s = 0.311$$

$\mu_s = 0.31$ ◀

(*b*) **Hawser Wrapped Three Turns around Bollard.** Using the value of μ_s obtained in part *a*,



$$\beta = 3(2\pi \text{ rad}) = 18.85 \text{ rad}$$

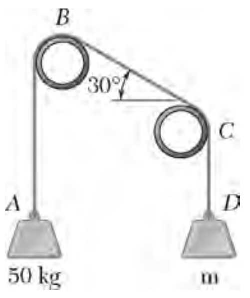
$$T_1 = 150 \text{ N} \quad \mu_s = 0.311$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

$$\frac{T_2}{150 \text{ N}} = e^{(0.311)(18.85)} = e^{5.862} = 351.5$$

$$T_2 = 52725 \text{ N}$$

$T_2 = 52.7 \text{ kN}$ ◀



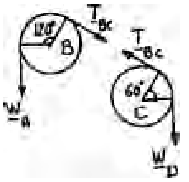
PROBLEM 8.102

A rope *ABCD* is looped over two pipes as shown. Knowing that the coefficient of static friction is 0.25, determine (*a*) the smallest value of the mass *m* for which equilibrium is possible, (*b*) the corresponding tension in portion *BC* of the rope.

SOLUTION

We apply Eq. (8.14) to pipe *B* and pipe *C*.

$$\frac{T_2}{T_1} = e^{\mu_s \beta} \qquad (8.14)$$



Pipe *B*: $T_2 = W_A, \quad T_1 = T_{BC}$

$$\mu_s = 0.25, \quad \beta = \frac{2\pi}{3}$$

$$\frac{W_A}{T_{BC}} = e^{0.25(2\pi/3)} = e^{\pi/6} \qquad (1)$$

Pipe *C*: $T_2 = T_{BC}, \quad T_1 = W_D, \quad \mu_s = 0.25, \quad \beta = \frac{\pi}{3}$

$$\frac{T_{BC}}{W_D} = e^{0.025(\pi/3)} = e^{\pi/12} \qquad (2)$$

(*a*) Multiplying Eq. (1) by Eq. (2):

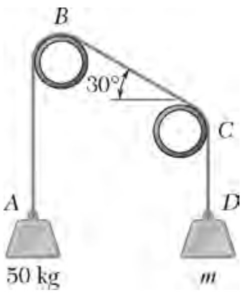
$$\frac{W_A}{W_D} = e^{\pi/6} \cdot e^{\pi/12} = e^{\pi/6 + \pi/12} = e^{\pi/4} = 2.193$$

$$W_D = \frac{W_A}{2.193} \qquad m = \frac{W_D}{g} = \frac{\frac{W_A}{g}}{2.193} = \frac{m_A}{2.193} = \frac{50 \text{ kg}}{2.193}$$

$$m = 22.8 \text{ kg} \blacktriangleleft$$

(*b*) From Eq. (1):

$$T_{BC} = \frac{W_A}{e^{\pi/6}} = \frac{(50 \text{ kg})(9.81 \text{ m/s}^2)}{1.688} = 291 \text{ N} \blacktriangleleft$$



PROBLEM 8.103

A rope *ABCD* is looped over two pipes as shown. Knowing that the coefficient of static friction is 0.25, determine (*a*) the largest value of the mass *m* for which equilibrium is possible, (*b*) the corresponding tension in portion *BC* of the rope.

SOLUTION

See *FB* diagrams of Problem 8.102. We apply Eq. (8.14) to pipes *B* and *C*.

Pipe *B*: $T_1 = W_A, \quad T_2 = T_{BC}, \quad \mu_s = 0.25, \quad \beta = \dfrac{2\pi}{3}$

$$\frac{T_2}{T_1} = e^{\mu_s \beta}: \quad \frac{T_{BC}}{W_A} = e^{0.25(2\pi/3)} = e^{\pi/6} \qquad (1)$$

Pipe *C*: $T_1 = T_{BC}, \quad T_2 = W_D, \quad \mu_s = 0.25, \quad \beta = \dfrac{\pi}{3}$

$$\frac{T_2}{T_1} = e^{\mu_s \beta}: \quad \frac{W_D}{T_{BC}} = e^{0.25(\pi/3)} = e^{\pi/12} \qquad (2)$$

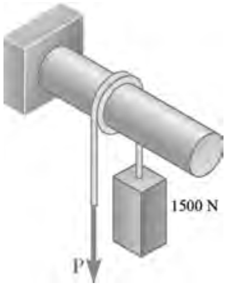
(*a*) Multiply Eq. (1) by Eq. (2):

$$\frac{W_D}{W_A} = e^{\pi/6} \cdot e^{\pi/12} = e^{\pi/6 + \pi/12} = e^{\pi/4} = 2.193$$

$$W_0 = 2.193 W_A \quad m = 2.193 m_A = 2.193(50 \text{ kg}) \qquad m = 109.7 \text{ kg} \blacktriangleleft$$

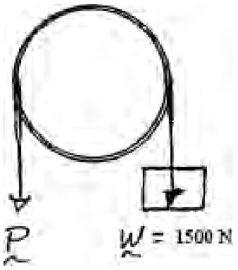
(*b*) From Eq. (1): $T_{BC} = W_A e^{\pi/6} = (50 \text{ kg})(9.81 \text{ m/s}^2)(1.688) = 828 \text{ N}$ ◀

PROBLEM 8.104



A 1500-N block is supported by a rope that is wrapped $1\frac{1}{2}$ times around a horizontal rod. Knowing that the coefficient of static friction between the rope and the rod is 0.15, determine the range of values of P for which equilibrium is maintained.

SOLUTION



$$\beta = 1.5 \text{ turns} = 3\pi \text{ rad}$$

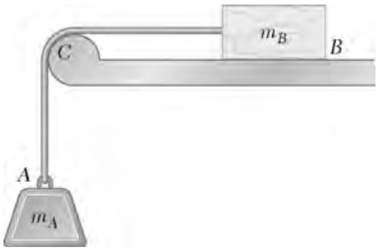
For impending motion of W up,

$$P = We^{\mu_s \beta} = (1500 \text{ N})e^{(0.15)3\pi}$$
$$= 6166.81 \text{ N}$$

For impending motion of W down,

$$P = We^{-\mu_s \beta} = (1500 \text{ N})e^{-(0.15)3\pi}$$
$$= 364.856 \text{ N}$$

For equilibrium, $365 \text{ N} \le P \le 6170 \text{ N}$ ◀

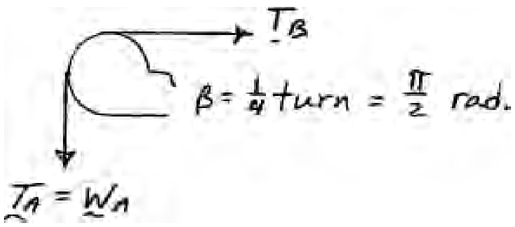


PROBLEM 8.105

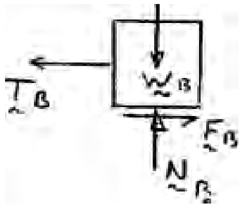
The coefficient of static friction between block B and the horizontal surface and between the rope and support C is 0.40. Knowing that $m_A = 12$ kg, determine the smallest mass of block B for which equilibrium is maintained.

SOLUTION

Support at C:



FBD block B:



$$W_A = mg = (12 \text{ kg})g$$

$$\uparrow \Sigma F_y = 0: \quad N_B - W_B = 0 \quad \text{or} \quad N_B = W_B$$

Impending motion: $\qquad F_B = \mu_s N_B = 0.4 N_B = 0.4 W_B$

$$\rightarrow \Sigma F_x = 0: \quad F_B - T_B = 0 \quad \text{or} \quad T_B = F_B = 0.4 W_B$$

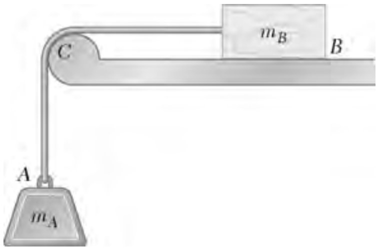
At support, for impending motion of W_A down:

$$W_A = T_B e^{\mu_s \beta}$$

so $\qquad T_B = W_A e^{-\mu_s \beta} = (12 \text{ kg}) g^{-(0.4)\pi/2} = (6.4019 \text{ kg})g$

Now $\qquad W_B = \frac{T_B}{0.4} = \left(\frac{6.4019 \text{ kg}}{0.4}\right) g = 16.0048 g$

so that $\qquad m_B = \frac{W_B}{g} = \frac{16.0048 g}{g}$ $\qquad m_B = 16.00 \text{ kg}$ ◀

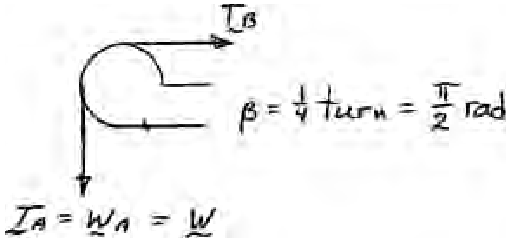


PROBLEM 8.106

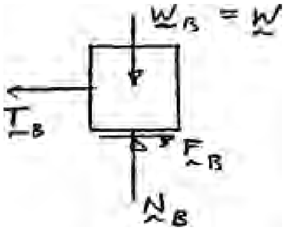
The coefficient of static friction μ_s is the same between block B and the horizontal surface and between the rope and support C. Knowing that $m_A = m_B$, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

Support at C:



FBD B:



$$\uparrow \Sigma F_y = 0: \quad N_B - W = 0 \quad \text{or} \quad N_B = W$$

Impending motion: $F_B = \mu_s N_B = \mu_s W$

$$\rightarrow \Sigma F_x = 0: \quad F_B - T_B = 0 \quad \text{or} \quad T_B = F_B = \mu_s W$$

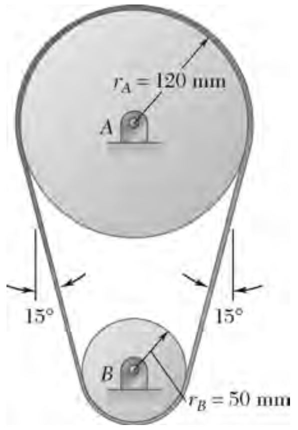
Impending motion of rope on support: $W = T_B e^{\mu_s \beta} = \mu_s W e^{\mu_s \beta}$

or $1 = \mu_s e^{\mu_s \beta}$

or $\mu_s e^{\pi/2 \mu_s} = 1$

Solving numerically: $\mu_s = 0.475$ ◄

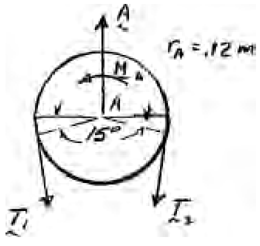
PROBLEM 8.107



A flat belt is used to transmit a couple from drum *B* to drum *A*. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest couple that can be exerted on drum *A*.

SOLUTION

FBD's drums:



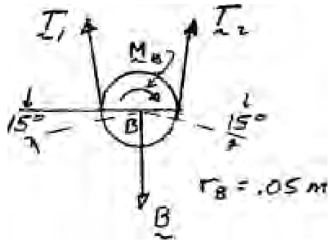
$$\beta_A = 180° + 30° = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

$$\beta_B = 180° - 30° = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Since $\beta_B < \beta_A$, slipping will impend first on *B* (friction coefficients being equal)

So

$$T_2 = T_{\max} = T_1 e^{\mu_s \beta_B}$$

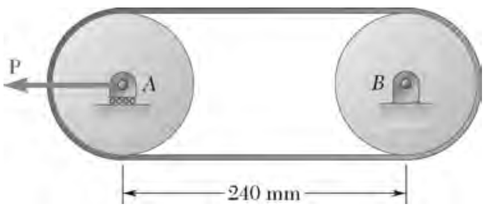


$$450\text{ N} = T_1 e^{(0.4)5\pi/6} \quad \text{or} \quad T_1 = 157.914\text{ N}$$

$$\Sigma M_A = 0: \quad M_A + (0.12\text{ m})(T_1 - T_2) = 0$$

$$M_A = (0.12\text{ m})(450\text{ N} - 157.914\text{ N}) = 35.05\text{ N}\cdot\text{m}$$

$$M_A = 35.1\text{ N}\cdot\text{m} \blacktriangleleft$$

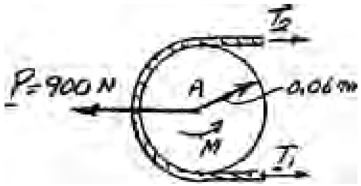


PROBLEM 8.108

A flat belt is used to transmit a couple from pulley A to pulley B. The radius of each pulley is 60 mm, and a force of magnitude $P = 900$ N is applied as shown to the axle of pulley A. Knowing that the coefficient of static friction is 0.35, determine (a) the largest couple that can be transmitted, (b) the corresponding maximum value of the tension in the belt.

SOLUTION

Drum A:



$$\frac{T_2}{T_1} = e^{\mu_s \pi} = e^{(0.35)\pi}$$

$$T_2 = 3.0028 T_1$$

$$\beta = 180° = \pi \text{ radians}$$

(a) Torque: $\circlearrowleft + \Sigma M_A = 0: \quad M - (675.15 \text{ N})(0.06 \text{ m}) + (224.84 \text{ N})(0.06 \text{ m})$

$$M = 27.0 \text{ N}\cdot\text{m} \blacktriangleleft$$

(b) $\xrightarrow{+} \Sigma F_x = 0: \quad T_1 + T_2 - 900 \text{ N} = 0$

$$T_1 + 3.0028 T_1 - 900 \text{ N} = 0$$

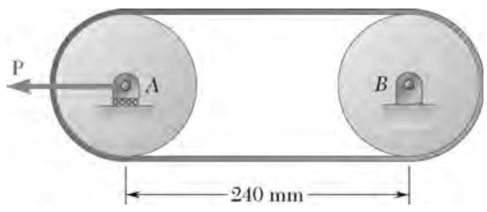
$$4.00282 T_1 = 900$$

$$T_1 = 224.841 \text{ N}$$

$$T_2 = 3.0028(224.841 \text{ N}) = 675.15 \text{ N}$$

$$T_{max} = 675 \text{ N} \blacktriangleleft$$

PROBLEM 8.109

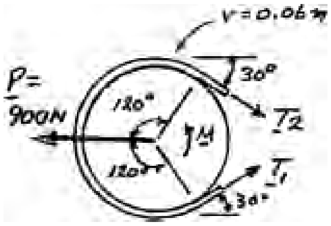


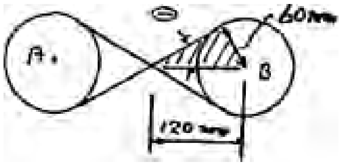
Solve Problem 8.108 assuming that the belt is looped around the pulleys in a figure eight.

PROBLEM 8.108 A flat belt is used to transmit a couple from pulley A to pulley B. The radius of each pulley is 60 mm, and a force of magnitude $P = 900$ N is applied as shown to the axle of pulley A. Knowing that the coefficient of static friction is 0.35, determine (a) the largest couple that can be transmitted, (b) the corresponding maximum value of the tension in the belt.

SOLUTION

<u>Drum A</u>:





$$\beta = 240° = 240°\frac{\pi}{180°} = \frac{4}{3}\pi$$

$$\sin\theta = \frac{60}{120} = \frac{1}{2}$$

$$\theta = 30°$$

$$\frac{T_2}{T_1} = e^{\mu_s\beta} = e^{0.35(4/3\pi)}$$

$$T_2 = 4.3322T_1$$

(a) <u>Torque</u>: $\circlearrowleft + \Sigma M_B = 0$: $M - (844.3\text{ N})(0.06\text{ m}) + (194.9\text{ N})(0.06\text{ m}) = 0$

$M = 39.0\text{ N}\cdot\text{m}$ ◀

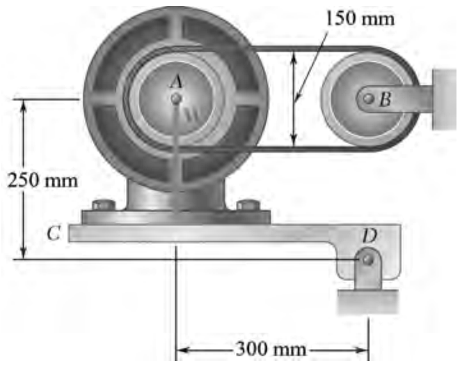
(b) $\xrightarrow{+} \Sigma F_x = 0$: $(T_1 + T_2)\cos 30° - 900\text{ N}$

$$(T_1 + 4.3322T_1)\cos 30° = 900$$

$$T_1 = 194.90\text{ N}$$

$$T_2 = 4.3322(194.90\text{ N}) = 844.3\text{ N}$$

$T_{max} = 844$ N ◀

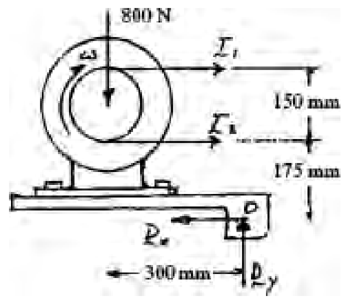


PROBLEM 8.110

In the pivoted motor mount shown the weight **W** of the 800 N motor is used to maintain tension in the drive belt. Knowing that the coefficient of static friction between the flat belt and drums *A* and *B* is 0.40, and neglecting the weight of platform *CD*, determine the largest couple that can be transmitted to drum *B* when the drive drum *A* is rotating clockwise.

SOLUTION

FBD motor and mount:



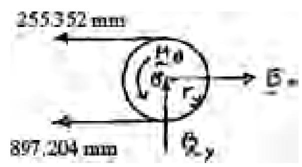
Impending belt slip: cw rotation

$$T_2 = T_1 e^{\mu_s \beta} = T_1 e^{0.40\pi} = 3.5136 T_1$$

$$\circlearrowright \Sigma M_D = 0: \quad (0.3\text{ m})(800\text{ N}) - (0.175\text{ m})T_2 - (0.325\text{ m})T_1 = 0$$

$$240\text{ N}\cdot\text{m} = [(0.175\text{ m})(3.5136) + 0.325\text{ m}]T_1$$

$$T_1 = 255.352\text{ N}, \quad T_2 = 3.5136 T_1 = 897.204\text{ N}$$



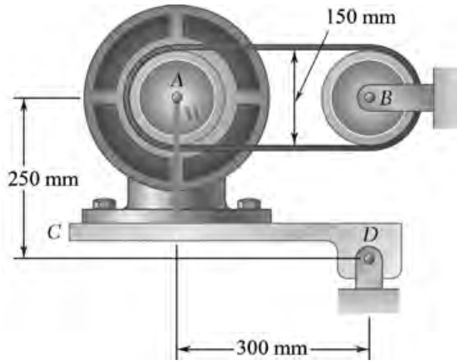
FBD drum at *B*:

$$\circlearrowright \Sigma M_B = 0: \quad M_B - (0.075\text{ m})(897.204 - 255.352)\text{N} = 0$$

$r = 75\text{ mm} = 0.075\text{ m}$

$$\Rightarrow M_B = 48.1389\text{ N}\cdot\text{m}$$

$M_B = 48.1\text{ N}\cdot\text{m}$ ◄



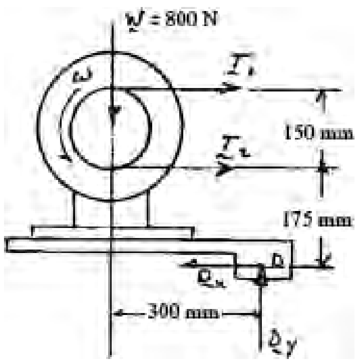
PROBLEM 8.111

Solve Problem 8.110 assuming that the drive drum *A* is rotating counterclockwise.

PROBLEM 8.110 In the pivoted motor mount shown the weight **W** of the 800-N motor is used to maintain tension in the drive belt. Knowing that the coefficient of static friction between the flat belt and drums *A* and *B* is 0.40, and neglecting the weight of platform *CD*, determine the largest couple that can be transmitted to drum *B* when the drive drum *A* is rotating clockwise.

SOLUTION

FBD motor and mount:



Impending belt slip: ccw rotation

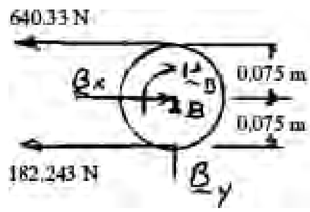
$$T_1 = T_2 e^{\mu_s \beta} = T_2 e^{0.40\pi} = 3.5136 T_2$$

$$\circlearrowleft \Sigma M_D = 0: \quad (0.3 \text{ m})(800 \text{ N}) - (0.325 \text{ m})T_1 - (0.175 \text{ m})T_2 = 0$$

$$240 \text{ N}\cdot\text{m} = [(0.325 \text{ m})(3.5136) + (0.175 \text{ m})]T_2 = 0$$

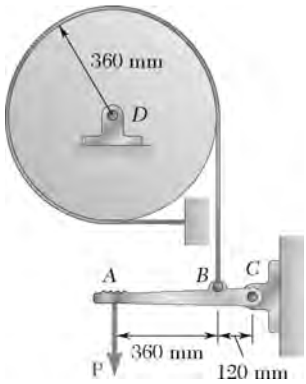
$$T_2 = 182.243 \text{ N}, \quad T_1 = 3.5136 T_2 = 640.330 \text{ N}$$

FBD drum at B:



$$\circlearrowleft \Sigma M_B = 0: \quad (0.075 \text{ m})(640.33 \text{ N} - 182.243 \text{ N}) - M_B = 0$$

$$M_B = 34.356 \text{ N}\cdot\text{m} \blacktriangleleft$$

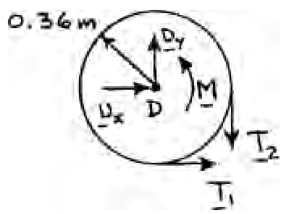


PROBLEM 8.112

A band brake is used to control the speed of a flywheel as shown. The coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.25$. Determine the magnitude of the couple being applied to the flywheel, knowing that $P = 45$ N and that the flywheel is rotating counterclockwise at a constant speed.

SOLUTION

Free body: Cylinder



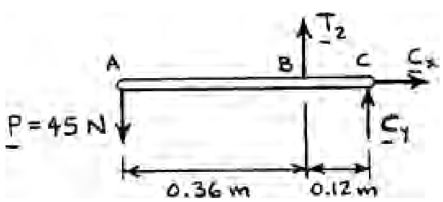
Since slipping of band relative to cylinder is clockwise, $\mathbf{T}_1$ and $\mathbf{T}_2$ are located as shown.

From free body: Lever *ABC*

$$\circlearrowleft^{+} \Sigma M_C = 0: \quad (45 \text{ N})(0.48 \text{ m}) - T_2(0.12 \text{ m}) = 0$$

$$T_2 = 180 \text{ N}$$

Free body: Lever *ABC*



From free body: Cylinder

Using Eq. (8.14) with $\mu_k = 0.25$ and $\beta = 270° = \dfrac{3\pi}{2}$ rad:

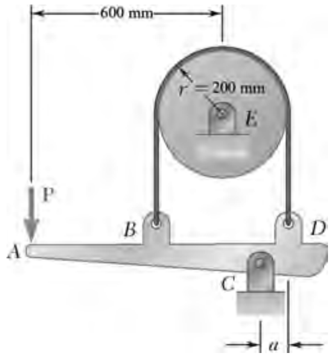
$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{(0.25)(3\pi/2)} = e^{3\pi/8}$$

$$T_1 = \frac{T_2}{e^{3\pi/8}} = \frac{180 \text{ N}}{3.2482} = 55.415 \text{ N}$$

$$\circlearrowleft^{+} \Sigma M_D = 0: \quad (55.415 \text{ N})(0.36 \text{ m}) - (180 \text{ N})(0.36 \text{ m}) + M = 0$$

$\mathbf{M} = 44.9 \text{ N}\cdot\text{m} \circlearrowleft$ ◀

PROBLEM 8.113

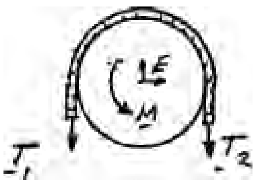


The speed of the brake drum shown is controlled by a belt attached to the control bar *AD*. A force **P** of magnitude 125 N is applied to the control bar at *A*. Determine the magnitude of the couple being applied to the drum, knowing that the coefficient of kinetic friction between the belt and the drum is 0.25, that a = 100 mm, and that the drum is rotating at a constant speed (*a*) counterclockwise, (*b*) clockwise.

SOLUTION

(*a*) <u>Counterclockwise rotation</u>

<u>Free body: Drum</u>

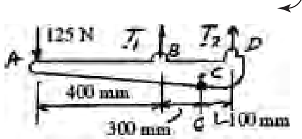


$$r = 200 \text{ mm} = 0.2 \text{ m} \quad \beta = 180° = \pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933 T_1$$

<u>Free body: Control bar</u>



$$+\circlearrowleft \Sigma M_C = 0: \quad T_1(0.3 \text{ mm}) - T_2(0.1 \text{ m}) - (125 \text{ N})(0.7 \text{ m}) = 0$$

$$T_1(0.3 \text{ m}) - 2.1933 T_1(0.1) - 87.5 = 0$$

$$T_1 = 1084.666 \text{ N}$$

$$T_2 = 2.1933(1084.666 \text{ N}) = 2378.998 \text{ N}$$

<u>Return to free body of drum</u>

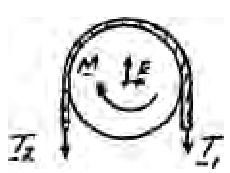
$$+\circlearrowleft \Sigma M_E = 0: \quad M + T_1(0.2 \text{ m}) - T_2(0.2 \text{ m}) = 0$$

$$M + 0.2 \text{ m}(1084.666 - 2378.998) \text{ N}$$

$$M = 258.866 \text{ N} \cdot \text{m}$$

$M = 259 \text{ N} \cdot \text{m}$ ◀

(*b*) <u>Clockwise rotation</u>



$$r = 200 \text{ mm} = 0.2 \text{ m} \quad \beta = \pi \text{ rad}$$

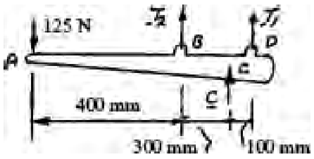
$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933 T_1$$

(Note T_2 & T_1 are opposite than the case in *a*)

PROBLEM 8.113 (Continued)

Free body: Control rod



$$\stackrel{+}{\circlearrowleft}\Sigma M_C = 0: \quad T_2(0.3\,\text{m}) - T_1(0.1\,\text{m}) - (125\text{ N})(0.7\text{ m}) = 0$$

$$2.1933T_1(0.3) - T_1(0.1) - 87.5 = 0$$

$$T_1 = 156.813\text{ N}$$

$$T_2 = 2.1933(156.813\text{ N})$$

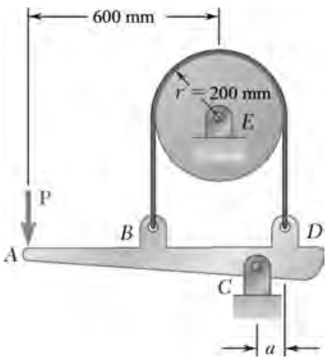
$$T_2 = 343.938\text{ N}$$

Return to free body of drum

$$\stackrel{+}{\circlearrowleft}\Sigma M_E = 0: \quad M + T_1(0.2\text{ m}) - T_2(0.2\text{ m}) = 0$$

$$M + (156.813\text{ N})(0.2\text{ m}) - (343.938)(0.2\text{ m}) = 0$$

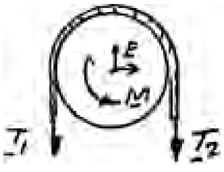
$$M = 37.425\,\text{N}\cdot\text{m} \qquad M = 37.4\text{ N}\cdot\text{m} \blacktriangleleft$$



PROBLEM 8.114

Knowing that $a = 100$ mm, determine the maximum value of the coefficient of static friction for which the brake is not self-locking when the drum rotates counterclockwise.

SOLUTION

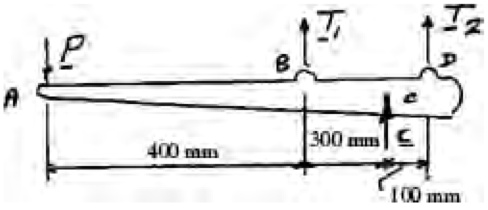


$r = 200\,\text{m}, \quad \beta = 180° = \pi$ radians

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{\mu_s \pi}$$

$$T_2 = e^{\mu_s \pi} T_1$$

Free body: Control rod



$$+\circlearrowleft \Sigma M_C = 0: \quad P(0.7\text{ m}) - T_1(0.3\text{m}) + T_2(0.1\text{ m}) = 0$$

$$0.7P - 0.3T_1 + e^{\mu\pi} T_1(0.1) = 0$$

For self-locking brake: $P = 0$

$$0.3T_1 = 0.1T_1 e^{\mu_s \pi}$$

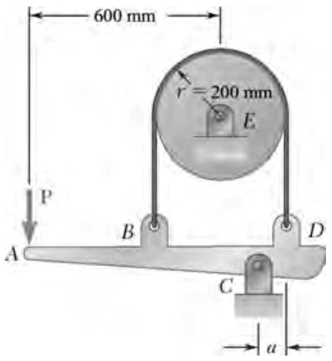
$$e^{\mu_s \pi} = 3$$

$$\mu_s \pi = \ln 3 = 1.0986$$

$$\mu_s = \frac{1.0986}{\pi} = 0.3497$$

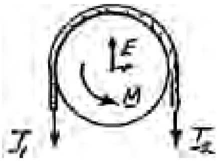
$\mu_s = 0.350$ ◀

PROBLEM 8.115



Knowing that the coefficient of static friction is 0.30 and that the brake drum is rotating counterclockwise, determine the minimum value of a for which the brake is not self-locking.

SOLUTION

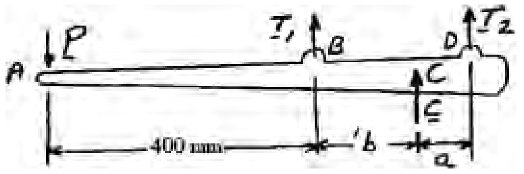


$r = 200$ mm, $\beta = \pi$ radians

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.30\pi} = 2.5663$$

$$T_2 = 2.5663 T_1$$

Free body: Control rod



$$b = 0.4\,\text{m} - a$$

$$+\circlearrowleft \Sigma M_C = 0: \quad P(0.4\text{ m} + b) - T_1 b + T_2 a = 0$$

For brake to be self-locking, $P = 0$

$$T_2 a = T_1 b; \quad 2.5663 T_1 a = T_1(0.4\,\text{m} - a)$$

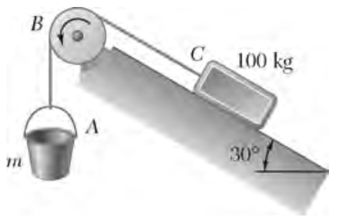
$$2.5663a = 0.4\,\text{m} - a$$

$$3.5663a = 0.4\,\text{m} \Rightarrow a = 0.112161\,\text{m}$$

or $a = 112.161\,\text{mm}$

$a = 112.2$ mm ◀

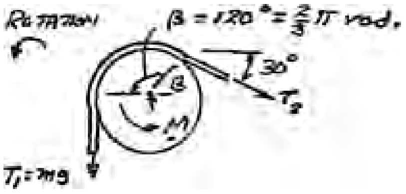
PROBLEM 8.116



Bucket A and block C are connected by a cable that passes over drum B. Knowing that drum B rotates slowly counterclockwise and that the coefficients of friction at all surfaces are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the smallest combined mass m of the bucket and its contents for which block C will (a) remain at rest, (b) start moving up the incline, (c) continue moving up the incline at a constant speed.

SOLUTION

Free body: Drum



$$\frac{T_2}{mg} = e^{\mu \frac{2}{3}\pi}$$

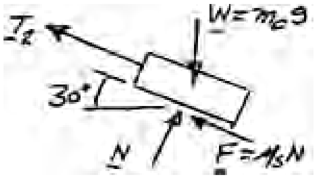
$$T_2 = mge^{2\mu\pi/3} \tag{1}$$

(a) Smallest m for block C to remain at rest

Cable slips on drum.

Eq. (1) with $\mu_k = 0.25$; $T_2 = mge^{2(0.25)\pi/3} = 1.6881mg$

Block C: At rest, motion impending↘



$$+\nearrow \Sigma F = 0: \quad N - m_C g\cos 30°$$

$$N = m_C g\cos 30°$$

$$F = \mu_s N = 0.35 m_C g\cos 30°$$

$$m_C = 100 \text{ kg}$$

$$+\nwarrow \Sigma F = 0: \quad T_2 + F - m_C g\sin 30° = C$$

$$1.6881mg + 0.35m_C g\cos 30° - m_C g\sin 30° = 0$$

$$1.6881m = 0.19689m_C$$

$$m = 0.11663m_C = 0.11663(100 \text{ kg}); \qquad m = 11.66 \text{ kg} \blacktriangleleft$$

(b) Smallest m to start block moving up

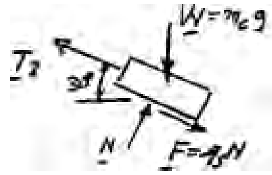
No slipping at both drum and block: $\mu_s = 0.35$

Eq. (1): $T_2 = mge^{2(0.35)\pi/3} = 2.0814\,mg$

PROBLEM 8.116 (Continued)

Block C:

Motion impending ↖



$$m_C = 100 \text{ kg}$$

$$+\nearrow \Sigma F = 0: \quad N - mg\cos 30°$$

$$N = m_C g \cos 30°$$

$$F = \mu_s N = 0.35 m_C g \cos 30°$$

$$+\nwarrow \Sigma F = 0: \quad T_2 - F - m_C g \sin 30° = 0$$

$$2.0814mg - 0.35 m_C g \cos 30° - m_C g \sin 30° = 0$$

$$2.0814m = 0.80311 m_C$$

$$m = 0.38585 m_C = 0.38585(100 \text{ kg})$$

$$m = 38.6 \text{ kg} \blacktriangleleft$$

(c) Smallest m to keep block moving up drum: No slipping: $\mu_s = 0.35$

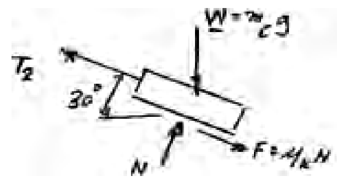
Eq. (1) with $\mu_s = 0.35$

$$T_2 = mg^{2\mu_s \pi/3} = mge^{2(0.35)\pi/3}$$

$$T_2 = 2.0814mg$$

Block C: Moving up plane, thus $\mu_k = 0.25$

Motion up ↖



$$+\nearrow \Sigma F = 0: \quad N - m_C g \cos 30° = 0$$

$$N = m_C g \cos 30°$$

$$F = \mu_k N = 0.25 m_C g \cos 30°$$

$$+\nwarrow \Sigma F = 0: \quad T_2 - F - m_C g \sin 30° = 0$$

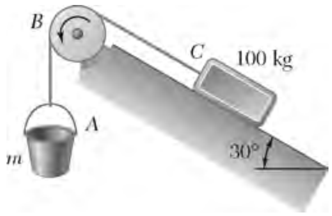
$$2.0814mg - 0.25 m_C g \cos 30° - m_C g \sin 30° = 0$$

$$2.0814m = 0.71651 m_C$$

$$m = 0.34424 m_C = 0.34424(100 \text{ kg})$$

$$m = 34.4 \text{ kg} \blacktriangleleft$$

PROBLEM 8.117

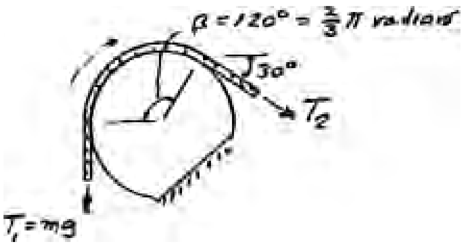


Solve Problem 8.116 assuming that drum B is frozen and cannot rotate.

PROBLEM 8.116 Bucket A and block C are connected by a cable that passes over drum B. Knowing that drum B rotates slowly counterclockwise and that the coefficients of friction at all surfaces are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the smallest combined mass m of the bucket and its contents for which block C will (*a*) remain at rest, (*b*) start moving up the incline, (*c*) continue moving up the incline at a constant speed.

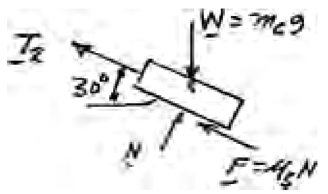
SOLUTION

(*a*) Block C remains at rest: Motion impends ↘



Drum:
$$\frac{T_2}{mg} = e^{\mu_k \beta} = e^{0.35(2\pi/3)}$$
$$T_2 = 2.0814mg$$

Block C: Motion impends ↘

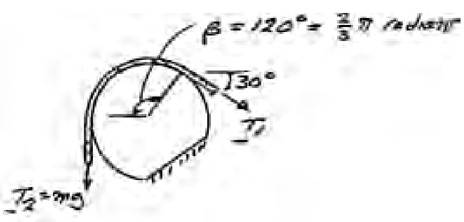


$$\nearrow \Sigma F = 0: \quad N - m_C g \cos 30° = 0$$
$$N = m_C g \cos 30°$$
$$F = \mu_s N = 0.35 m_C g \cos 30°$$
$$+\nwarrow \Sigma F = 0: \quad T_2 + F - m_C g \sin 30° = 0$$
$$2.0814mg + 0.35 m_C g \cos 30° - m_C g \sin 30° = 0$$
$$2.0814mg = 0.19689 m_C$$
$$m = 0.09459 m_C = 0.09459(100 \text{ kg})$$

$m = 9.46$ kg ◀

(*b*) Block C: Starts moving up $\mu_s = 0.35$

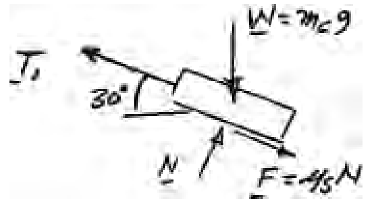
Drum: Impending motion of cable ↻



$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$
$$\frac{mg}{T_1} = e^{0.35(2/3\pi)}$$
$$T_1 = \frac{mg}{2.0814}$$
$$= 0.48045mg$$

PROBLEM 8.117 (Continued)

Block C: Motion impends $\nwarrow$



$+\nearrow \Sigma F = 0$: $N - m_C g \cos 30°$

$$N = m_C g \cos 30°$$
$$F = \mu_s N = 0.35 m_C g \cos 30°$$

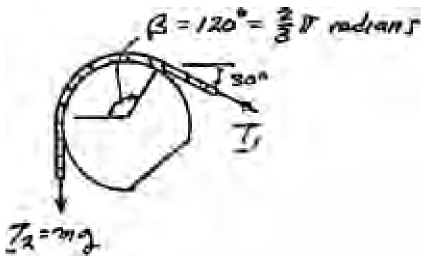
$+\nwarrow \Sigma F = 0$: $T_1 - F - m_C g \sin 30° = 0$

$$0.48045 mg - 0.35 m_C g \cos 30° - 0.5 m_C g = 0$$
$$0.48045 m = 0.80311 m_C$$
$$m = 1.67158 m_C = 1.67158(100 \text{ kg})$$

$m = 167.2$ kg ◀

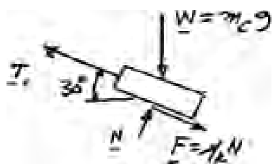
(*c*) Smallest m to keep block moving $\nwarrow$

Drum: Motion of cable $\circlearrowleft$



$$\mu_k = 0.25$$
$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25(2/3\pi)}$$
$$\frac{mg}{T_1} = 1.6881$$
$$T_1 = \frac{mg}{1.6881} = 0.59238 mg$$

Block C: Block moves $\nwarrow$



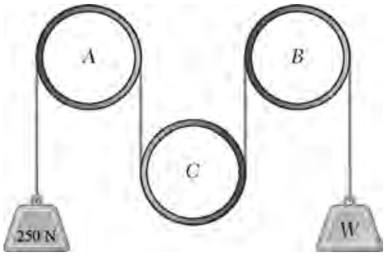
$+\nearrow \Sigma F = 0$: $N - m_C g \cos 30° = 0$

$$N = m_C g \cos 30°$$
$$F = \mu_k N = 0.25 m_C g \cos 30°$$

$+\nwarrow \Sigma F = 0$: $T_1 - F - m_C g \sin 30° = 0$

$$0.59238 mg - 0.25 m_C g \cos 30° - 0.5 m_C g = 0$$
$$0.59238 m = 0.71651 m_C$$
$$m = 1.20954 m_C = 1.20954(100 \text{ kg})$$

$m = 121.0$ kg ◀



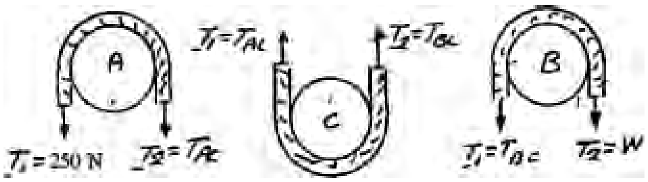
PROBLEM 8.118

A cable is placed around three parallel pipes. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (*a*) the smallest weight *W* for which equilibrium is maintained, (*b*) the largest weight *W* that can be raised if pipe *B* is slowly rotated counterclockwise while pipes *A* and *C* remain fixed.

SOLUTION

(*a*) <u>Smallest *W* for equilibrium</u> For smallest *W*, impending motion is that of *W* moving upwards.

$$B = \pi, \quad \mu = \mu_s$$



$$\frac{250\text{ N}}{T_{AC}} = e^{0.25\pi} \qquad \frac{T_{AC}}{T_{BC}} = e^{0.25\pi} \qquad \frac{T_{BC}}{W} = e^{0.25\pi}$$

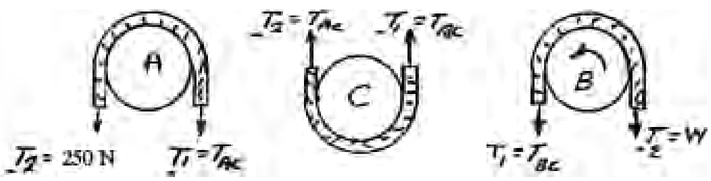
$$\frac{250\text{ N}}{T_{AC}} \cdot \frac{T_{AC}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/4} \cdot e^{\pi/4} \cdot e^{\pi/4} = e^{3\pi/4} = 10.551$$

$$W = \frac{250\text{ N}}{10.551} = 23.694\text{ N};$$

$W = 23.7$ N ◀

(*b*) <u>Largest *W* which can be raised by pipe *B* rotated</u> ↺

$$\beta = \pi, \mu = \mu_k \qquad \beta = \pi, \mu = \mu_k \qquad \beta = \pi, \mu = \mu_s$$



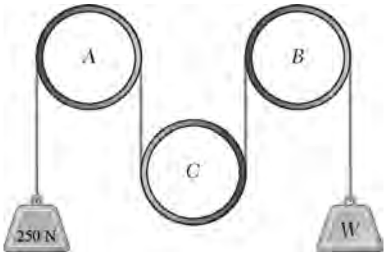
$$\frac{250\text{ N}}{T_{AC}} = e^{0.2\pi} \qquad \frac{T_{AC}}{T_{BC}} = e^{0.2\pi} \qquad \frac{W}{T_{BC}} = e^{0.25\pi}$$

$$\frac{250\text{ N}}{T_{AC}} \cdot \frac{T_{AC}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/5} \cdot e^{\pi/5} \cdot e^{-\pi/4} = e^{\pi(1/5+1/5-1/4)}$$

$$= e^{3\pi/20} = 1.602$$

$$\frac{250\text{ N}}{W} = 1.602; \qquad W = \frac{250\text{ N}}{1.602} = 156.05\text{ N}$$

$W = 156.1$ N ◀



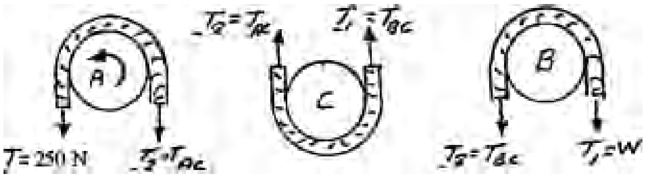
PROBLEM 8.119

A cable is placed around three parallel pipes. Two of the pipes are fixed and do not rotate; the third pipe is slowly rotated. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the largest weight W that can be raised (*a*) if only pipe A is rotated counterclockwise, (*b*) if only pipe C is rotated clockwise.

SOLUTION

(*a*) Pipe A rotates

$\beta = \pi, \mu = \mu_s$ $\quad$ $\beta = \pi, \mu = \mu_k$ $\quad$ $\beta = \pi, \mu = \mu_k$



$$\frac{T_{AC}}{250\text{ N}} = e^{0.25\pi} \qquad \frac{T_{AC}}{T_{BC}} = e^{0.2\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi}$$

$$\frac{T_{AC}}{250\text{ N}} \cdot \frac{T_{BC}}{T_{AC}} \cdot \frac{W}{T_{BC}} = e^{\pi/4} \cdot e^{-\pi/5} \cdot e^{-\pi/5}$$

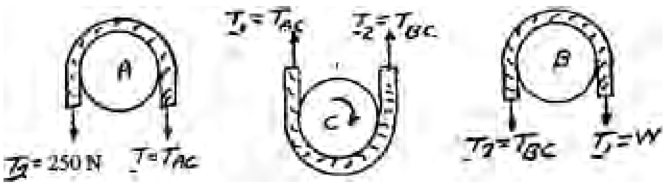
$$= e^{\pi(1/4-1/5-1/5)} = e^{-3\pi/20} = 0.62423$$

$$\frac{W}{250\text{ N}} = 0.62423; \quad W = 156.0575\text{ N}$$

$W = 156.1\text{ N}$ ◀

(*b*) Pipe C rotates

$\beta = \pi, \mu = \mu_k$ $\quad$ $\beta = \pi, \mu = \mu_s$ $\quad$ $\beta = \pi, \mu = \mu_k$



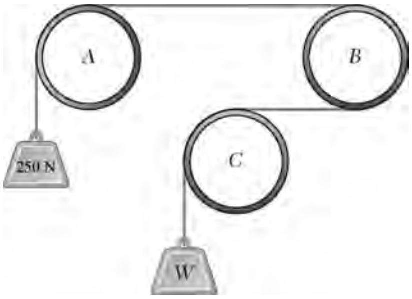
$$\frac{250\text{ N}}{T_{AC}} = e^{0.2\pi} \qquad \frac{T_{BC}}{T_{AC}} = e^{0.25\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi}$$

$$\frac{250\text{ N}}{T_{AC}} \cdot \frac{T_{AC}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/5} \cdot e^{-\pi/4} \cdot e^{\pi/5} = e^{\pi(1/5-1/4+1/5)} = e^{3\pi/20}$$

$$\frac{250\text{ N}}{W} = e^{3\pi/20} = 1.602$$

$$W = \frac{250\text{ N}}{1.602} = 156.0575\text{ N}$$

$W = 156.1\text{ N}$ ◀

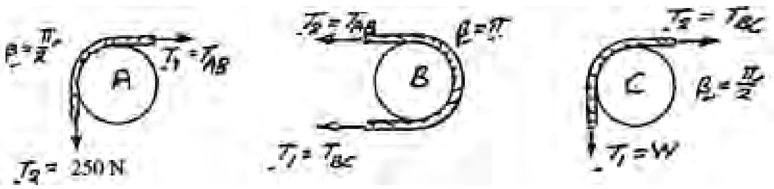


PROBLEM 8.120

A cable is placed around three parallel pipes. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (*a*) the smallest weight *W* for which equilibrium is maintained, (*b*) the largest weight *W* that can be raised if pipe *B* is slowly rotated counterclockwise while pipes *A* and *C* remain fixed.

SOLUTION

(*a*) Impending motion for smallest *W* is that of *W* moving upwards. $\mu = \mu_s = 0.25$ at all pipes.



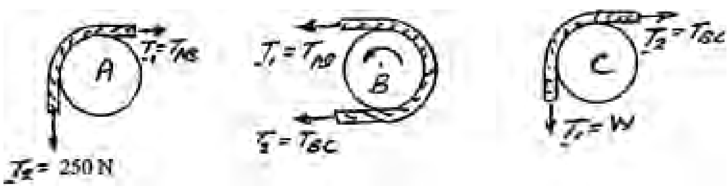
$$\frac{250\text{ N}}{T_{AB}} = e^{0.25\pi/2} \qquad \frac{T_{AB}}{T_{BC}} = e^{0.25\pi} \qquad \frac{T_{BC}}{W} = e^{0.25\pi/2}$$

$$\frac{250\text{ N}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/8} \cdot e^{\pi/4} \cdot e^{\pi/8} = e^{\pi/8+\pi/4+\pi/8} = e^{\pi/2} = 4.8105$$

$$\frac{250\text{ N}}{W} = 4.8015; \quad W = 52.067\text{ N}$$

$W = 52.1$ N ◀

(*b*) Pipe *B* rotated ↺ $\quad \beta = \frac{\pi}{2};\ \mu = \mu_k \qquad \beta = \pi;\ \mu = \mu_s \qquad \beta = \frac{\pi}{2};\ \mu = \mu_k$



$$\frac{250\text{ N}}{T_{AB}} = e^{0.2\pi/2} \qquad \frac{T_{BC}}{T_{AB}} = e^{0.25\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi/2}$$

$$\frac{250\text{ N}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/10} \cdot e^{-\pi/4} \cdot e^{\pi/10}$$

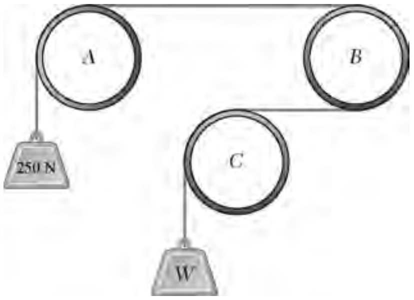
$$= e^{\pi/10-\pi/4+\pi/10} = e^{-\pi/20} = 0.85464$$

$$\frac{250\text{ N}}{W} = 0.85464$$

$$W = \frac{250\text{ N}}{0.85464} = 292.52\text{ N}$$

$W = 293$ N ◀

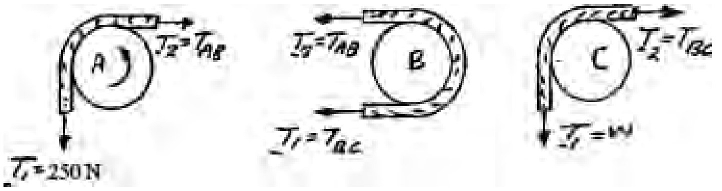
PROBLEM 8.121



A cable is placed around three parallel pipes. Two of the pipes are fixed and do not rotate; the third pipe is slowly rotated. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the largest weight W that can be raised (*a*) if only pipe A is rotated counterclockwise, (*b*) if only pipe C is rotated clockwise.

SOLUTION

(*a*) Pipe A rotates $\circlearrowleft$ $\qquad \beta = \dfrac{\pi}{2};\ \mu = \mu_s \qquad \beta = \pi,\ \mu = \mu_k \qquad \beta = \dfrac{\pi}{2};\ \mu = \mu_k$



$$\frac{T_{AB}}{250\text{ N}} = e^{0.25\pi/2} \qquad \frac{T_{AB}}{T_{BC}} = e^{0.2\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi/2}$$

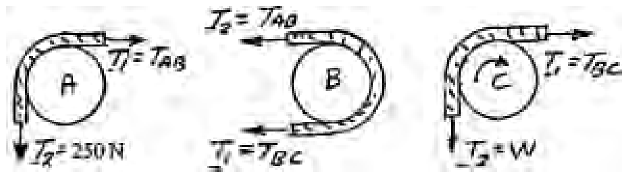
$$\frac{T_{AB}}{250\text{ N}} \cdot \frac{T_{BC}}{T_{AB}} \cdot \frac{W}{T_{BC}} = e^{\pi/8} \cdot e^{-\pi/5} \cdot e^{-\pi/10}$$

$$= e^{\pi(1/8-1/5-1/10)} = e^{-7\pi/40} = 0.57708$$

$$\frac{W}{250\text{ N}} = 0.57708; \quad W = 144.27\text{ N}$$

$W = 144.3$ N ◀

(*b*) Pipe C rotates $\circlearrowright$ $\qquad \beta = \dfrac{\pi}{2};\ \mu = \mu_k \quad \beta = \pi;\ \mu = \mu_k \quad \beta = \dfrac{\pi}{2};\ \mu = \mu_s$



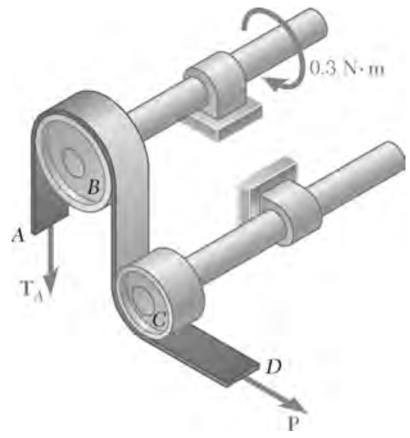
$$\frac{250\text{ N}}{T_{AB}} = e^{0.2\pi/2} \qquad \frac{T_{AB}}{T_{BC}} = e^{0.2\pi} \qquad \frac{W}{T_{BC}} = e^{0.25\pi/2}$$

$$\frac{250\text{ N}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/10} \cdot e^{\pi/5} \cdot e^{-\pi/8} = e^{7\pi/40} = 0.57708$$

$$\frac{250\text{ N}}{W} = 0.57708$$

$$W = 144.27\text{ N}$$

$W = 144.3$ N ◀

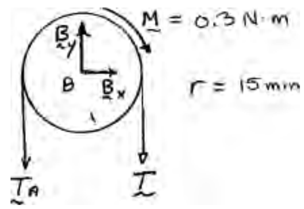


PROBLEM 8.122

A recording tape passes over the 20-mm-radius drive drum B and under the idler drum C. Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

SOLUTION

FBD drive drum:



$$\circlearrowleft \Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

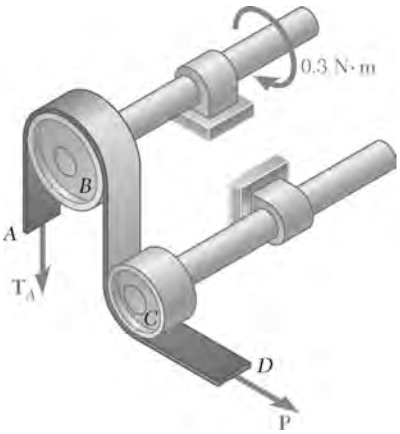
$$T_A - T = \frac{M}{r} = \frac{300 \text{ N} \cdot \text{mm}}{20 \text{ mm}} = 15.0000 \text{ N}$$

Impending slipping: $\quad T_A = Te^{\mu_s \beta} = Te^{0.4\pi}$

So $\quad T(e^{0.4\pi} - 1) = 15.0000 \text{ N}$

or $\quad T = 5.9676 \text{ N}$

If C is free to rotate, $P = T$ $\quad P = 5.97 \text{ N}$ ◀



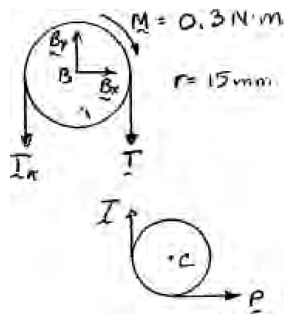
PROBLEM 8.123

Solve Problem 8.122 assuming that the idler drum C is frozen and cannot rotate.

PROBLEM 8.122 A recording tape passes over the 20-mm-radius drive drum B and under the idler drum C. Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

SOLUTION

FBD drive drum:



$$\Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

$$T_A - T = \frac{M}{r} = 300 \text{ N} \cdot \text{mm} = 15.0000 \text{ N}$$

Impending slipping: $T_A = Te^{\mu_s \beta} = Te^{0.4\pi}$

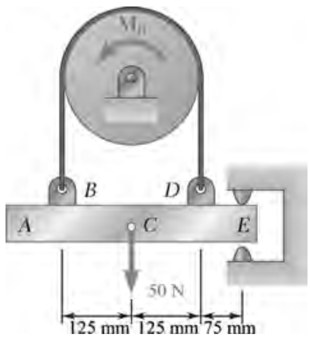
So $(e^{0.4\pi} - 1)T = 15.000 \text{ N}$

or $T = 5.9676 \text{ N}$

If C is fixed, the tape must slip

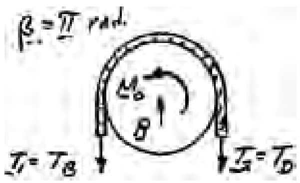
So $P = Te^{\mu_k \beta_C} = (5.9676 \text{ N})e^{0.3\pi/2} = 9.5600 \text{ N}$ $\qquad P = 9.56 \text{ N}$ ◀

PROBLEM 8.124



The 50 N bar AE is suspended by a cable that passes over a 125-mm-radius drum. Vertical motion of end E of the bar is prevented by the two stops shown. Knowing that $\mu_s = 0.30$ between the cable and the drum, determine (*a*) the largest counterclockwise couple $\mathbf{M}_0$ that can be applied to the drum if slipping is not to occur, (*b*) the corresponding force exerted on end E of the bar.

SOLUTION

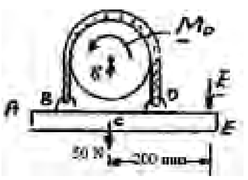


Drum: Slipping impends $\qquad \mu_s = 0.30$

$$\frac{T_2}{T_1} = e^{\mu\beta}: \quad \frac{T_D}{T_B} = e^{0.30\pi} = 2.5663$$

$$T_D = 2.5663T_B$$

(*a*) Free-body: Drum and bar



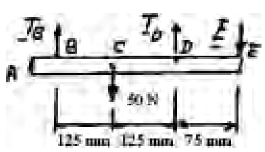
$$\overset{+}{\circlearrowleft}\Sigma M_C = 0: \quad M_0 - E(200 \text{ mm}) = 0 \qquad (1)$$

$$M_0 = (200 \text{ mm})E = 3783.53 \text{ N}\cdot\text{mm}$$

$$= 3.7835 \text{ N}\cdot\text{m}$$

$$M_0 = 3.78 \text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

(*b*) Bar *AE*:



$$+\!\uparrow \Sigma F_y = 0: \quad T_B + T_D - E - 50 \text{ N} = 0$$

$$T_B + 2.5663T_B - E - 50 \text{ N} = 0$$

$$3.5663T_B - E - 50 \text{ N} = 0$$

$$E = 3.5663T_B - 50 \text{ N} \qquad (2)$$

$$\overset{+}{\circlearrowleft}\Sigma M_D = 0: \quad E(75 \text{ mm}) - (50 \text{ N})(125 \text{ mm}) + T_B(250 \text{ mm}) = 0$$

$$(3.5663T_B - 50 \text{ N})(75 \text{ mm}) - 6250 \text{ N}\cdot\text{mm} + T_B(250 \text{ mm}) = 0$$

$$517.4725T_B = 10{,}000 \quad T_B = 19.3247 \text{ N}$$

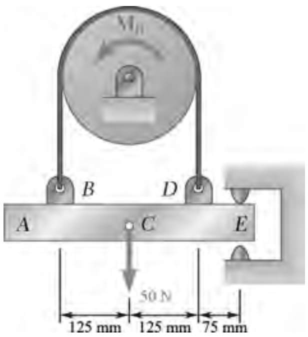
Eq. (2):

$$E = 3.5663(19.3247) - 50 \text{ N}$$

$$E = +18.9177 \text{ N}$$

$$\mathbf{E} = 18.9 \text{ N} \downarrow \blacktriangleleft$$

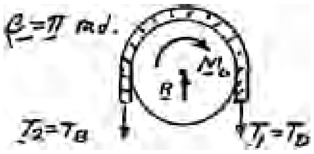
PROBLEM 8.125



Solve Problem 8.124 assuming that a clockwise couple $\mathbf{M}_0$ is applied to the drum.

PROBLEM 8.124 The 50-N bar *AE* is suspended by a cable that passes over a 125 mm-radius drum. Vertical motion of end *E* of the bar is prevented by the two stops shown. Knowing that $\mu_s = 0.30$ between the cable and the drum, determine (*a*) the largest counterclockwise couple $\mathbf{M}_0$ that can be applied to the drum if slipping is not to occur, (*b*) the corresponding force exerted on end *E* of the bar.

SOLUTION



Drum: Slipping impends

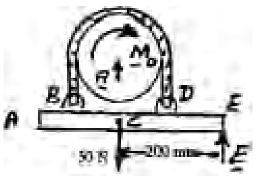
$$\mu_s = 0.30$$

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$\frac{T_B}{T_D} = e^{0.30\pi} = 2.5663$$

$$T_B = 2.5663 T_D$$

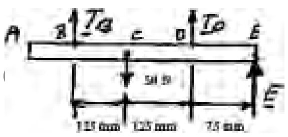
(*a*) Free body: Drum and bar



$$+\circlearrowleft \Sigma M_C = 0: \quad M_0 - E(200\,\text{mm}) = 0 \text{ w} \qquad (1)$$

$$M_0(200\,\text{mm})(E) = 200 \times 10.7707 = 2154.14\ \text{N}\cdot\text{mm}$$
$$= 2.15414\ \text{N}\cdot\text{m}$$

$$\mathbf{M}_0 = 2.15\ \text{N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



$$+\uparrow \Sigma F_y = 0: \quad T_B + T_D + E - 50\ \text{N} = 0$$

$$= 2.5663 T_D + T_D + E - 50\ \text{N}$$

$$E = -3.5663 T_D + 50\ \text{N} \qquad (2)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad T_D(250\ \text{mm}) - (50\ \text{N})(125\ \text{mm}) + E(325\ \text{mm}) = 0$$

$$T_D(250\,\text{mm}) - 6250\ \text{N}\cdot\text{mm} + (-3.5663 T_D + 50\ \text{N})(325\ \text{mm}) = 0$$

$$-909.0475 T_D + 10000\ \text{N}\cdot\text{mm} = 0; \quad T_D = 11.00\ \text{N}$$

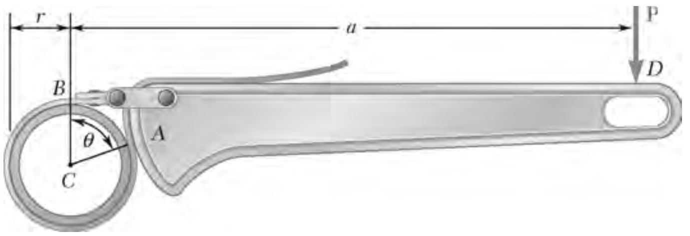
Eq. (2):

$$E = -3.5663(11.00\ \text{N}) + 50$$

$$E = 10.7707\ \text{N} \qquad \mathbf{E} = 10.77\ \text{N} \uparrow \blacktriangleleft$$

PROBLEM 8.126

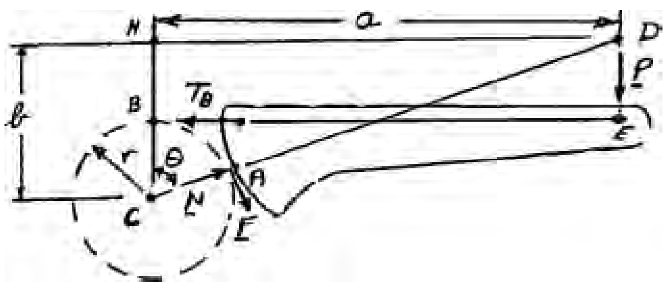
The strap wrench shown is used to grip the pipe firmly without marring the external surface of the pipe. Knowing that the coefficient of static friction is the same for all surfaces of contact, determine the smallest value of μ_s for which the wrench will be self-locking when $a = 200$ mm, $r = 30$ mm, and $\theta = 65°$.



SOLUTION

For wrench to be self-locking $(P = 0)$, the value of μ_s must prevent slipping of strap which is in contact with the pipe from Point A to Point B and must be large enough so that at Point A the strap tension can increase from zero to the minimum tension required to develop "belt friction" between strap and pipe.

Free body: Wrench handle



Geometry In CDH:

$$CH = \frac{a}{\tan\theta}$$

$$CD = \frac{a}{\sin\theta}$$

$$DE = BH = CH - BC$$

$$DE = \frac{a}{\tan\theta} - r$$

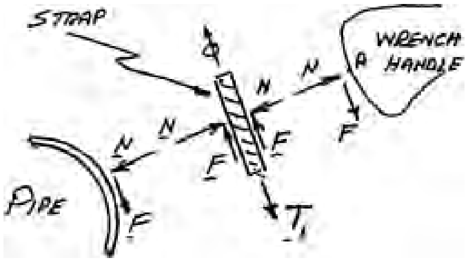
$$AD = CD - CA = \frac{a}{\sin\theta} - r$$

On wrench handle $\quad +\circlearrowleft \Sigma M_D = 0\colon \quad T_B(DE) - F(AD) = 0$

$$\frac{T_B}{F} = \frac{AD}{DE} = \frac{\dfrac{a}{\sin\theta} - r}{\dfrac{a}{\tan\theta} - r} \qquad (1)$$

PROBLEM 8.126 (Continued)

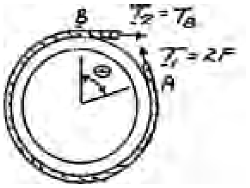
Free body: Strap at Point A



$$+\searrow \Sigma F = 0: \quad T_1 - 2F = 0$$

$$T_1 = 2F \tag{2}$$

Pipe and strap



$$\beta = (2\pi - \theta) \text{ radians}$$

Eq. (8.13):

$$\mu_s \beta = \ln \frac{T_2}{T_1}$$

$$\mu_s = \frac{1}{\beta} \ln \frac{T_B}{2F} \tag{3}$$

Return to free body of wrench handle

$$\xrightarrow{+} \Sigma F_x = 0: \quad N \sin\theta + F\cos\theta - T_B = 0$$

$$\frac{N}{F}\sin\theta = \frac{T_B}{F} - \cos\theta$$

Since $F = \mu_s N$, we have

$$\frac{1}{\mu_s}\sin\theta = \frac{T_B}{F} - \cos\theta$$

or

$$\mu_s = \frac{\sin\theta}{\frac{T_B}{F} - \cos\theta} \tag{4}$$

(*Note*: For a given set of data, we seek the larger of the values of μ_s from Eqs. (3) and (4).)

For $a = 200 \text{ mm}, \quad r = 30 \text{ mm}, \quad \theta = 65°$

Eq. (1):

$$\frac{T_B}{F} = \frac{\frac{200 \text{ mm}}{\sin 65°} - 30 \text{ mm}}{\frac{200 \text{ mm}}{\tan 65°} - 30 \text{ mm}}$$

$$= \frac{190.676 \text{ mm}}{63.262 \text{ mm}} = 3.0141$$

$$\beta = 2\pi - \theta = 2\pi - 65° \frac{\pi}{180°} = 5.1487 \text{ radians}$$

PROBLEM 8.126 (Continued)

Eq. (3):
$$\mu_s = \frac{1}{5.1487 \text{ rad}} \ln \frac{3.0141}{2}$$
$$= \frac{0.41015}{5.1487}$$
$$= 0.0797 \quad \triangleleft$$

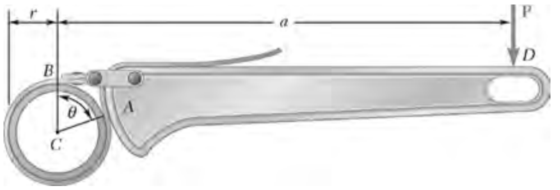
Eq. (4):
$$\mu_s = \frac{\sin 65°}{3.0141 - \cos 65°}$$
$$= \frac{0.90631}{2.1595}$$
$$= 0.3497 \quad \triangleleft$$

We choose the larger value: $\mu_s = 0.350$ ◀

PROBLEM 8.127

Solve Problem 8.126 assuming that $\theta = 75°$.

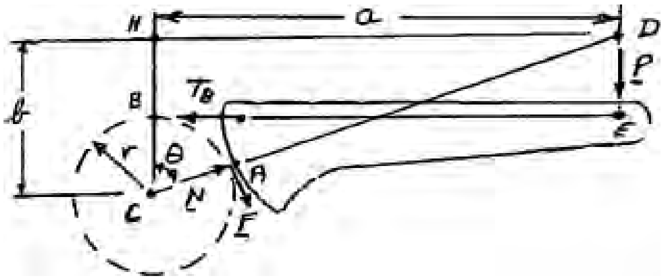
PROBLEM 8.126 The strap wrench shown is used to grip the pipe firmly without marring the external surface of the pipe. Knowing that the coefficient of static friction is the same for all surfaces of contact, determine the smallest value of μ_s for which the wrench will be self-locking when $a = 200$ mm, $r = 30$ mm, and $\theta = 65°$.



SOLUTION

For wrench to be self-locking $(P = 0)$, the value of μ_s must prevent slipping of strap which is in contact with the pipe from Point A to Point B and must be large enough so that at Point A the strap tension can increase from zero to the minimum tension required to develop "belt friction" between strap and pipe.

Free body: Wrench handle



Geometry In ΔCDH:

$$CH = \frac{a}{\tan\theta}$$

$$CD = \frac{a}{\sin\theta}$$

$$DE = BH = CH - BC$$

$$DE = \frac{a}{\tan\theta} - r$$

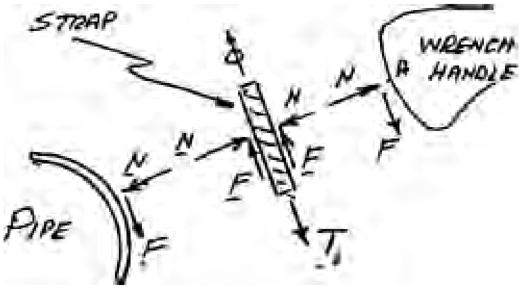
$$AD = CD - CA = \frac{a}{\sin\theta} - r$$

On wrench handle $\circlearrowleft\!+\ \Sigma M_D = 0\text{:}\quad T_B(DE) - F(AD) = 0$

$$\frac{T_B}{F} = \frac{AD}{DE} = \frac{\dfrac{a}{\sin\theta} - r}{\dfrac{a}{\tan\theta} - r} \tag{1}$$

PROBLEM 8.127 (Continued)

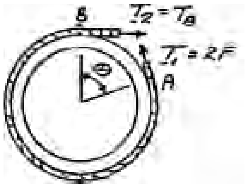
Free body: Strap at Point A



$$\overset{+}{\searrow} \Sigma F = 0: \quad T_1 - 2F = 0$$

$$T_1 = 2F \qquad (2)$$

Pipe and strap



$$\beta = (2\pi - \theta) \text{ radians}$$

Eq. (8.13):
$$\mu_s \beta = \ln \frac{T_2}{T_1}$$

$$\mu_s = \frac{1}{\beta} \ln \frac{T_B}{2F} \qquad (3)$$

Return to free body of wrench handle

$$\xrightarrow{+} \Sigma F_x = 0: \quad N \sin\theta + F \cos\theta - T_B = 0$$

$$\frac{N}{F} \sin\theta = \frac{T_B}{F} - \cos\theta$$

Since $F = \mu_s N$, we have
$$\frac{1}{\mu_s} \sin\theta = \frac{T_B}{F} - \cos\theta$$

or
$$\mu_s = \frac{\sin\theta}{\frac{T_B}{F} - \cos\theta} \qquad (4)$$

(*Note:* For a given set of data, we seek the larger of the values of μ_s from Eqs. (3) and (4).)

For
$$a = 200 \text{ mm}, \quad r = 30 \text{ mm}, \quad \theta = 75°$$

Eq. (1):
$$\frac{T_B}{F} = \frac{\frac{200 \text{ mm}}{\sin 75°} - 30 \text{ mm}}{\frac{200 \text{ mm}}{\tan 75°} - 30 \text{ mm}}$$

$$= \frac{177.055 \text{ mm}}{23.590 \text{ mm}} = 7.5056$$

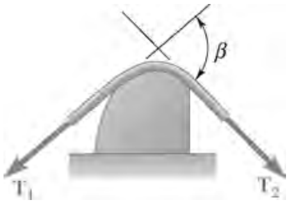
$$\beta = 2\pi - \theta = 2\pi - 75° \frac{\pi}{180°} = 4.9742$$

PROBLEM 8.127 (Continued)

Eq. (3):
$$\mu_s = \frac{1}{4.9742\ \text{rad}} \ln \frac{7.5056}{2}$$
$$= \frac{1.3225}{4.9742}$$
$$= 0.2659 \quad \triangleleft$$

Eq. (4):
$$\mu_s = \frac{\sin 75^\circ}{7.5056 - \cos 75^\circ}$$
$$= \frac{0.96953}{7.2468}$$
$$= 0.1333 \quad \triangleleft$$

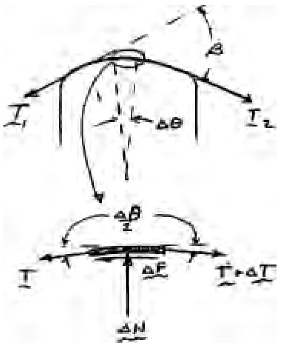
We choose the larger value: $\mu_s = 0.266$ ◀



PROBLEM 8.128

Prove that Eqs. (8.13) and (8.14) are valid for any shape of surface provided that the coefficient of friction is the same at all points of contact.

SOLUTION



$$\uparrow \Sigma F_n = 0: \quad \Delta N - [T + (T + \Delta T)] \sin\frac{\Delta\theta}{2} = 0$$

or
$$\Delta N = (2T + \Delta T)\sin\frac{\Delta\theta}{2}$$

$$\rightarrow \Sigma F_t = 0: \quad [(T + \Delta T) - T]\cos\frac{\Delta\theta}{2} - \Delta F = 0$$

or
$$\Delta F = \Delta T \cos\frac{\Delta\theta}{2}$$

Impending slipping:
$$\Delta F = \mu_s \Delta N$$

So
$$\Delta T \cos\frac{\Delta\theta}{2} = \mu_s 2T \sin\frac{\Delta\theta}{2} + \mu_s \Delta T \frac{\sin\Delta\theta}{2}$$

In limit as
$$\Delta\theta \to 0: \quad dT = \mu_s T d\theta \quad \text{or} \quad \frac{dT}{T} = \mu_s d\theta$$

So
$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^\beta \mu_s d\theta$$

and
$$\ln\frac{T_2}{T_1} = \mu_s\beta \qquad \text{or } T_2 = T_1 e^{\mu_s\beta} \blacktriangleleft$$

(*Note:* Nothing above depends on the shape of the surface, except it is assumed to be a smooth curve.)

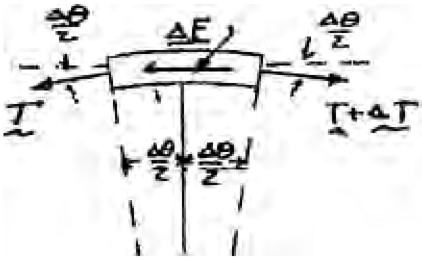
PROBLEM 8.129

Complete the derivation of Eq. (8.15), which relates the tension in both parts of a V belt.

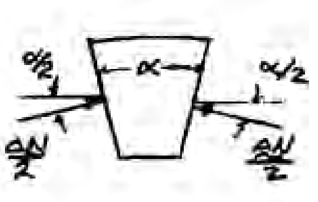
SOLUTION

Small belt section:

Side view:



End view:



$$\uparrow \Sigma F_y = 0: \quad 2\frac{\Delta N}{2}\sin\frac{\alpha}{2} - [T + (T + \Delta T)]\sin\frac{\Delta\theta}{2} = 0$$

$$\rightarrow \Sigma F_x = 0: \quad [(T + \Delta T) - T]\cos\frac{\Delta\theta}{2} - \Delta F = 0$$

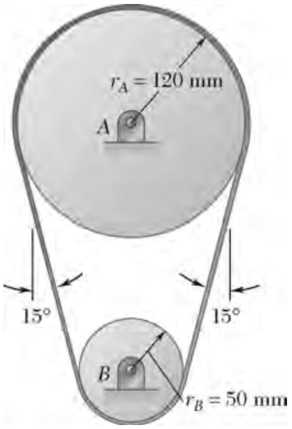
Impending slipping: $\quad \Delta F = \mu_s \Delta N \qquad \Delta T\cos\frac{\Delta\theta}{2} = \mu_s \dfrac{2T + \Delta T}{\sin\frac{\alpha}{2}}\sin\frac{\Delta\theta}{2}$

In limit as $\quad \Delta\theta \to 0: \quad dT = \dfrac{\mu_s T d\theta}{\sin\frac{\alpha}{2}} \quad$ or $\quad \dfrac{dT}{T} = \dfrac{\mu_s}{\sin\frac{\alpha}{2}}d\theta$

so
$$\int_{T_1}^{T_2}\frac{dT}{T} = \frac{\mu_s}{\sin\frac{\alpha}{2}}\int_0^\beta d\theta$$

or
$$\ln\frac{T_2}{T_1} = \frac{\mu_s\beta}{\sin\frac{\alpha}{2}}$$

or
$$T_2 = T_1 e^{\mu_s\beta/\sin\frac{\alpha}{2}} \blacktriangleleft$$



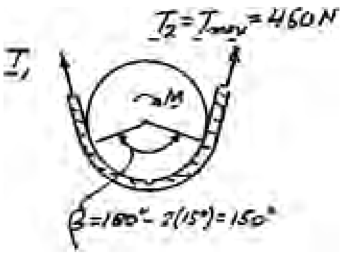
PROBLEM 8.130

Solve Problem 8.107 assuming that the flat belt and drums are replaced by a V belt and V pulleys with $\alpha = 36°$. (The angle α is as shown in Figure 8.15*a*.)

PROBLEM 8.107 A flat belt is used to transmit a couple from drum *B* to drum *A*. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest couple that can be exerted on drum *A*.

SOLUTION

Since β is smaller for pulley *B*. The belt will slip first at *B*.



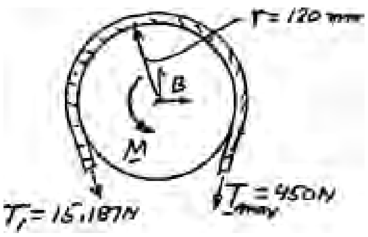
$$\beta = 150°\left(\frac{\pi \text{ rad}}{180°}\right) = \frac{5}{6}\pi \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta / \sin\frac{\alpha}{2}}$$

$$\frac{450 \text{ N}}{T_1} = e^{(0.4)\frac{5}{6}\pi/\sin 18°} = e^{3.389}$$

$$\frac{450 \text{ N}}{T_1} = 29.63; \quad T_1 = 15.187 \text{ N}$$

<u>Torque on pulley *A*</u>:



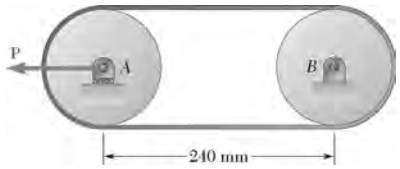
$$\circlearrowleft + \Sigma M_B = 0: \quad M - (T_{\text{max}} - T_1)(0.12 \text{ m}) = 0$$

$$M - (450 \text{ N} - 15.187 \text{ N})(0.12 \text{ m}) = 0$$

$$M = 52.18 \text{ N}\cdot\text{m}$$

$$M = 52.2 \text{ N}\cdot\text{m} \blacktriangleleft$$

PROBLEM 8.131

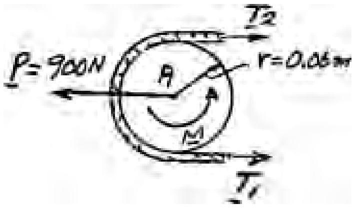


Solve Problem 8.108 assuming that the flat belt and pulleys are replaced by a V belt and V pulleys with $\alpha = 36°$. (The angle α is as shown in Figure 8.15*a*.)

PROBLEM 8.108 A flat belt is used to transmit a couple from pulley *A* to pulley *B*. The radius of each pulley is 60 mm, and a force of magnitude $P = 900$ N is applied as shown to the axle of pulley *A*. Knowing that the coefficient of static friction is 0.35, determine (*a*) the largest couple that can be transmitted, (*b*) the corresponding maximum value of the tension in the belt.

SOLUTION

Pulley *A*: $\beta = \pi$ rad



$$\frac{T_2}{T_1} = e^{\mu_s \beta / \sin\frac{\alpha}{2}}$$

$$\frac{T_2}{T_1} = e^{0.35\pi / \sin 18°}$$

$$\frac{T_2}{T_1} = e^{3.558} = 35.1$$

$$T_2 = 35.1 T_1$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad T_1 + T_2 - 900 \text{ N} = 0$$

$$T_1 + 35.1T_1 - 900 \text{ N} = 0$$

$$T_1 = 24.93 \text{ N}$$

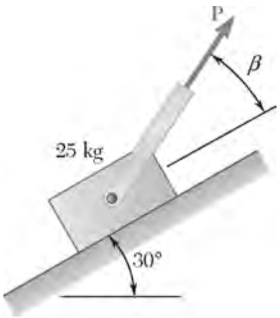
$$T_2 = 35.1(24.93 \text{ N}) = 875.03 \text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M - T_2(0.06 \text{ m}) + T_1(0.06 \text{ m}) = 0$$

$$M - (875.03 \text{ N})(0.06 \text{ m}) + (24.93 \text{ N})(0.06 \text{ m}) = 0$$

$$M = 51.0 \text{ N} \cdot \text{m} \blacktriangleleft$$

$$T_{max} = T_2 \qquad T_{max} = 875 \text{ N} \blacktriangleleft$$

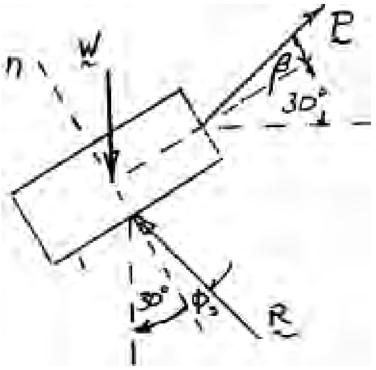


PROBLEM 8.132

Knowing that the coefficient of friction between the 25-kg block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of *P* required to start the block moving up the incline, (*b*) the corresponding value of β.

SOLUTION

FBD block (Impending motion up)

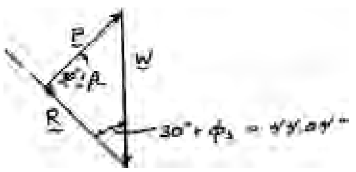


$$W = mg$$
$$= (25\text{ kg})(9.81\text{ m/s}^2)$$
$$= 245.25\text{ N}$$

$$\phi_s = \tan^{-1}\mu_s$$
$$= \tan^{-1}(0.25)$$
$$= 14.04°$$

(*a*) (*Note:* For minimum *P*, $\mathbf{P} \perp \mathbf{R}$ so $\beta = \phi_s$.)

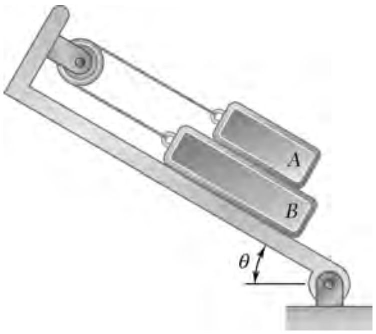
Then



$$P = W\sin(30° + \phi_s)$$
$$= (245.25\text{ N})\sin 44.04°$$

$P_{min} = 170.5$ N ◀

(*b*) We have $\beta = \phi_s$ $\beta = 14.04°$ ◀



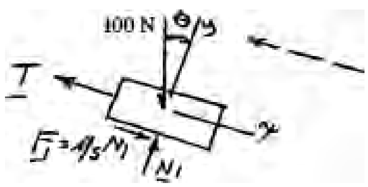
PROBLEM 8.133

The 100-N block A and the 150-N block B are supported by an incline that is held in the position shown. Knowing that the coefficient of static friction is 0.15 between all surfaces of contact, determine the value of θ for which motion is impending.

SOLUTION

Since motion impends, $F = \mu_s N$ at all surfaces.

<u>Free body: Block A</u>



Impending motion:

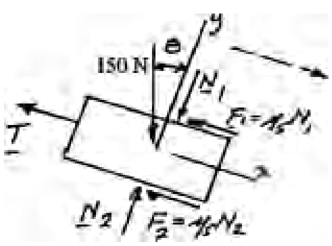
$$\Sigma F_y = 0: \quad N_1 = 100\cos\theta$$

$$+\Sigma F_x = 0: \quad T - 100\sin\theta - \mu_s N_1 = 0$$

$$T = 100\sin\theta + 0.15(100\cos\theta)$$

$$T = 100\sin\theta + 15\cos\theta \qquad (1)$$

<u>Free body: Block B</u>



Impending motion:

$$+\Sigma F_y = 0: \quad N_2 - 150\cos\theta - N_1 = 0$$

$$N_2 = 150\cos\theta + 100\cos\theta = 250\cos\theta$$

$$F_2 = \mu_s N_2 = 0.15(250\cos\theta) = 37.5\cos\theta$$

$$+\Sigma F_x = 0: \quad T - 150\sin\theta + \mu_s N_1 + \mu_s N_2 = 0$$

$$T = 150\sin\theta - 0.15(100\cos\theta) - 0.15(250\cos\theta)$$

$$T = 150\sin\theta - 15\cos\theta - 37.5\cos\theta \qquad (2)$$

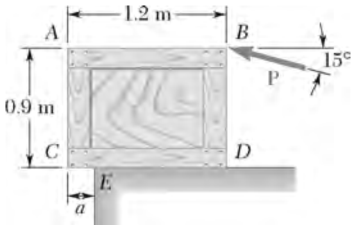
Eq. (1) subtracted by Eq. (2):

$$100\sin\theta + 15\cos\theta - 150\sin\theta + 15\cos\theta + 37.5\cos\theta = 0$$

$$67.5\cos\theta = 50\sin\theta, \quad \tan\theta = \frac{67.5}{50}$$

$\theta = 53.5°$ ◀

PROBLEM 8.134



A worker slowly moves a 50-kg crate to the left along a loading dock by applying a force **P** at corner *B* as shown. Knowing that the crate starts to tip about the edge *E* of the loading dock when *a* = 200 mm, determine (*a*) the coefficient of kinetic friction between the crate and the loading dock, (*b*) the corresponding magnitude *P* of the force.

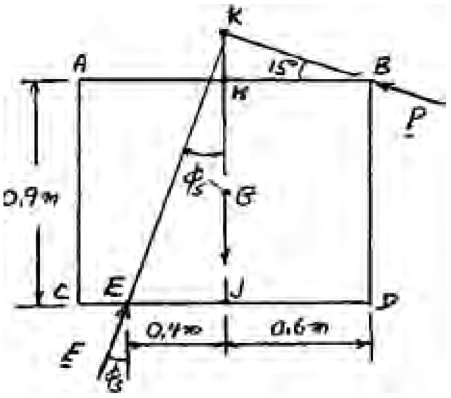
SOLUTION

Free body: Crate Three-force body.

Reaction *E* must pass through *K* where **P** and **W** intersect.

Geometry:

(*a*) $$HK = (0.6\text{ m})\tan 15° = 0.16077\text{ m}$$



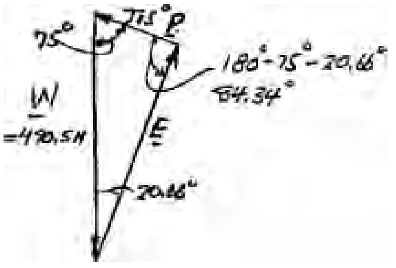
$$JK = 0.9\text{ m} + HK = 1.06077\text{ m}$$

$$\tan\phi_s = \frac{0.4\text{ m}}{1.06077\text{ m}} = 0.37708$$

$\mu_s = \tan\phi_s = 0.377$ ◀

$$\phi_s = 20.66°$$

Force triangle:



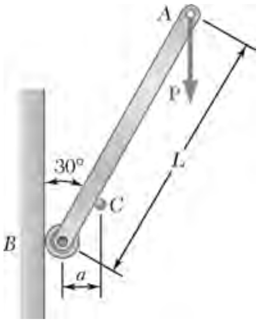
(*b*) $$W = (50\text{ kg})(9.81\text{ m/s}) = 490.5\text{ N}$$

Law of sines:

$$\frac{P}{\sin 20.66°} = \frac{490.5\text{ N}}{\sin 84.34°}$$

$$P = 173.91\text{ N}$$

$P = 173.9\text{ N}$ ◀



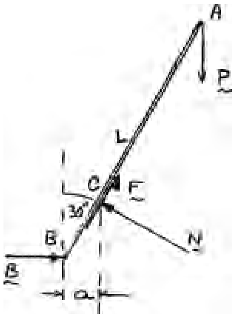
PROBLEM 8.135

A slender rod of length L is lodged between peg C and the vertical wall and supports a load **P** at end A. Knowing that the coefficient of static friction between the peg and the rod is 0.15 and neglecting friction at the roller, determine the range of values of the ratio L/a for which equilibrium is maintained.

SOLUTION

FBD rod:

Free-body diagram: (1) For motion of B impending upward:



$$+\circlearrowleft \Sigma M_B = 0: \quad PL\sin\theta - N_C\left(\frac{a}{\sin\theta}\right) = 0$$

$$N_C = \frac{PL}{a}\sin^2\theta \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_C\sin\theta - \mu_s N_C\cos\theta - P = 0$$

$$N_C(\sin\theta - \mu\cos\theta) = P$$

Substitute for N_C from Eq. (1), and solve for a/L.

$$\frac{a}{L} = \sin^2\theta(\sin\theta - \mu_s\cos\theta) \qquad (2)$$

For $\theta = 30°$ and $\mu_s = 0.15$:

$$\frac{a}{L} = \sin^2 30°(\sin 30° - 0.15\cos 30°)$$

$$\frac{a}{L} = 0.092524 \qquad \frac{L}{a} = 10.808$$

For motion of B impending downward, reverse sense of friction force F_C. To do this we make

$\mu_s = -0.15$ in. Eq. (2).

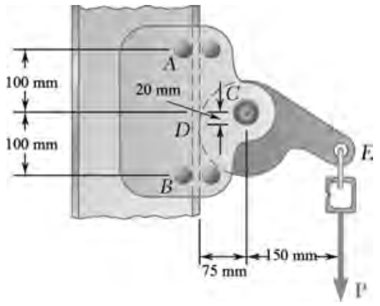
Eq. (2):

$$\frac{a}{L} = \sin^2 30°(\sin 30° - (-0.15)\cos 30°)$$

$$\frac{a}{L} = 0.15748 \qquad \frac{L}{a} = 6.350$$

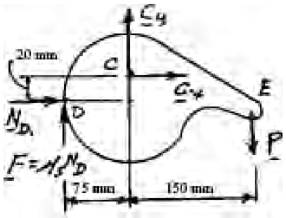
Range of values of L/a for equilibrium: $6.35 \le \dfrac{L}{a} \le 10.81$ ◀

PROBLEM 8.136



A safety device used by workers climbing ladders fixed to high structures consists of a rail attached to the ladder and a sleeve that can slide on the flange of the rail. A chain connects the worker's belt to the end of an eccentric cam that can be rotated about an axle attached to the sleeve at *C*. Determine the smallest allowable common value of the coefficient of static friction between the flange of the rail, the pins at *A* and *B*, and the eccentric cam if the sleeve is not to slide down when the chain is pulled vertically downward.

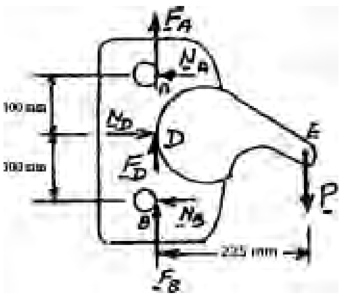
SOLUTION



Free body: Cam

$$+\circlearrowleft \Sigma M_C = 0: \quad N_D(20 \text{ mm}) - \mu_s N_D(75 \text{ mm}) - P(150 \text{ mm}) = 0$$

$$N_D = \frac{150P}{20 - 75\mu_s} \tag{1}$$



Free body: Sleeve and cam

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_D - N_A - N_B = 0$$

$$N_A + N_B = N_D \tag{2}$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_B + F_D - P = 0$$

or $$\mu_s(N_A + N_B + N_D) = P \tag{3}$$

Substitute from Eq. (2) into Eq. (3): $$\mu_s(2N_D) = P \qquad N_D = \frac{P}{2\mu_s} \tag{4}$$

Equate expressions for N_D from Eq. (1) and Eq. (4):

$$\frac{P}{2\mu_s} = \frac{150P}{20 - 75\mu_s}; \quad 20 - 75\mu_s = 300\mu_s$$

$$\mu_s = \frac{20}{375} \qquad \mu_s = 0.0533 \blacktriangleleft$$

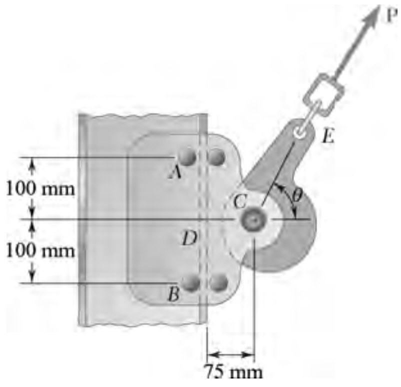
(*Note:* To verify that contact at pins *A* and *B* takes places as assumed, we shall check that $N_A > 0$ and $N_B > 0$.)

PROBLEM 8.136 (Continued)

From Eq. (4):
$$N_D = \frac{P}{2\mu_s} = \frac{P}{2(0.0533)} = 9.375P$$

From free body of cam and sleeve:
$$+\circlearrowleft \Sigma M_B = 0: \quad N_A(200\,\text{mm}) - N_D(100\text{ mm}) - P(225\text{ mm}) = 0$$
$$200N_A = (9.375P)(100) + 225P$$
$$N_A = 5.8125P > 0 \quad \text{OK}$$

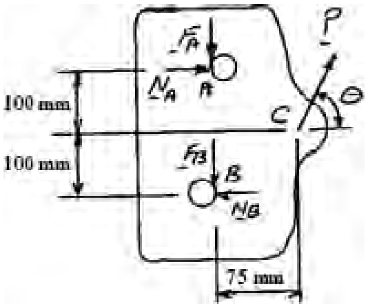
From Eq. (2):
$$N_A + N_B = N_D$$
$$5.8125P + N_B = 9.375P$$
$$N_B = 3.5625P > 0 \quad \text{OK}$$



PROBLEM 8.137

To be of practical use, the safety sleeve described in the preceding problem must be free to slide along the rail when pulled upward. Determine the largest allowable value of the coefficient of static friction between the flange of the rail and the pins at A and B if the sleeve is to be free to slide when pulled as shown in the figure, assuming (*a*) $\theta = 60°$, (*b*) $\theta = 50°$, (*c*) $\theta = 40°$.

SOLUTION



Note the cam is a two-force member.

Free body: Sleeve

We assume contact between rail and pins as shown.

$$+\circlearrowleft \Sigma M_C = 0: \quad F_A(75 \text{ mm}) + F_B(75 \text{ mm}) - N_A(100 \text{ mm}) - N_B(100 \text{ mm}) = 0$$

But $\qquad F_A = \mu_s N_A$

$\qquad\qquad F_B = \mu_s N_B$

We find $\qquad 75\mu_s(N_A + N_B) - 100(N_A + N_B) = 0$

$$\mu_s = \frac{100}{75} = 1.33333$$

We now verify that our assumption was correct.

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B + P\cos\theta = 0$$

$$N_B - N_A = P\cos\theta \qquad (1)$$

$$+\!\uparrow \Sigma F_y = 0: \quad -F_A - F_B + P\sin\theta = 0$$

$$\mu_s N_A + \mu_s N_B = P\sin\theta$$

$$N_A + N_B = \frac{P\sin\theta}{\mu_s} \qquad (2)$$

Add Eqs. (1) and (2): $\qquad 2N_B = P\left(\cos\theta + \dfrac{\sin\theta}{\mu_s}\right) > 0 \quad \text{OK}$

PROBLEM 8.137 (Continued)

Subtract Eq. (1) from Eq. (2): $$2N_A = P\left(\frac{\sin\theta}{\mu_s} - \cos\theta\right)$$

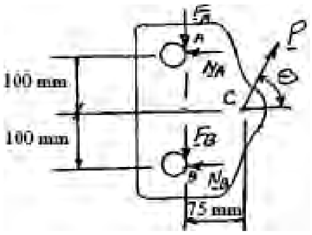
$N_A > 0$ only if $$\frac{\sin\theta}{\mu_s} - \cos\theta > 0$$

$$\tan\theta > \mu_s = 1.33333$$
$$\theta = 53.130°$$

(*a*) For case (*a*): Condition is satisfied, contact takes place as shown. Answer is correct.

$\mu_s = 1.333$ ◀

But for (*b*) and (*c*): $\theta < 53.130°$ and our assumption is wrong, N_A is directed to left.



$$\xrightarrow{+} \Sigma F_x = 0: \quad -N_A - N_B + P\cos\theta = 0$$

$$N_A + N_B = P\cos\theta \qquad (3)$$

$$+\!\uparrow \Sigma F_y = 0: \quad -F_A - F_A + P\sin\theta = 0$$

$$\mu_s(N_A + N_B) = P\sin\theta \qquad (4)$$

Divide Eq. (4) by Eq. (3):

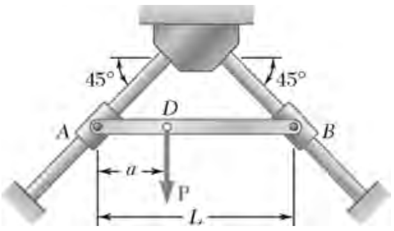
$$\mu_s = \tan\theta \qquad (5)$$

(*b*) We make $\theta = 50°$ in Eq. (5):

$$\mu_s = \tan 50°$$ $\mu_s = 1.192$ ◀

(*c*) We make $\theta = 40°$ in Eq. (5):

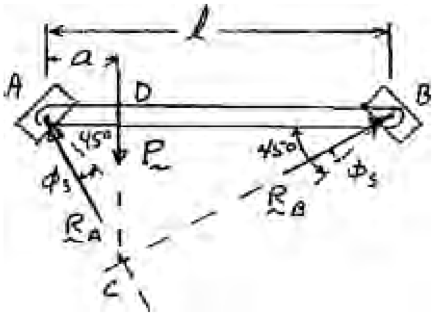
$$\mu_s = \tan 40°$$ $\mu_s = 0.839$ ◀



PROBLEM 8.138

Bar *AB* is attached to collars that can slide on the inclined rods shown. *A* force **P** is applied at Point *D* located at a distance *a* from end *A*. Knowing that the coefficient of static friction μ_s between each collar and the rod upon which it slides is 0.30 and neglecting the weights of the bar and of the collars, determine the smallest value of the ratio *a*/*L* for which equilibrium is maintained.

SOLUTION



FBD bar and collars:

Impending motion: $\phi_s = \tan^{-1}\mu_s$

$$= \tan^{-1} 0.3$$

$$= 16.6992°$$

Neglect weights: 3-force *FBD* and $\measuredangle ACB = 90°$

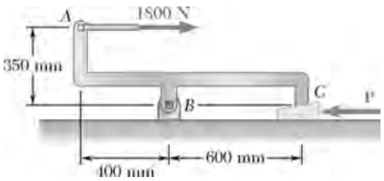
so $$AC = \frac{a}{\cos(45° + \phi_s)}$$

$$= l\sin(45° - \phi_s)$$

$$\frac{a}{l} = \sin(45° - 16.6992°)\cos(45° + 16.6992°)$$

$$\frac{a}{l} = 0.225 \blacktriangleleft$$

PROBLEM 8.139

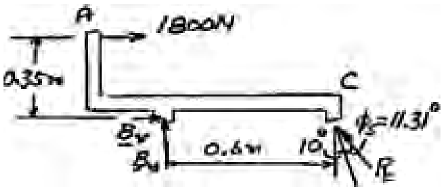


The machine part *ABC* is supported by a frictionless hinge at *B* and a 10° wedge at *C*. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine (*a*) the force **P** required to move the wedge, (*b*) the components of the corresponding reaction at *B*.

SOLUTION

$$\phi_s = \tan^{-1} 0.20 = 11.31°$$

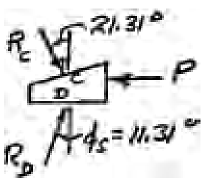
<u>Free body: Part *ABC*</u>



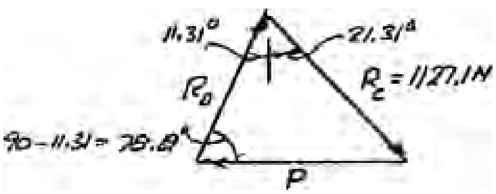
$$\overset{+}{\circlearrowleft} \Sigma M_B = 0 \quad (1800\text{ N})(0.35\text{ m}) - R\cos 21.31°(0.6\text{ m}) = 0$$

$$R_C = 1127.1\text{ N}$$

<u>Free body: Wedge</u>



<u>Force triangle:</u>



(*a*) <u>Law of sines</u>:

$$\frac{P}{\sin(11.31° + 21.31°)} = \frac{1127.1\text{ N}}{\sin 78.69°}$$

$$P = 619.6\text{ N} \qquad \mathbf{P} = 620\text{ N} \longleftarrow ◀$$

(*b*) Return to part *ABC*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 1800\text{ N} - R_C \sin 21.31° = 0$$

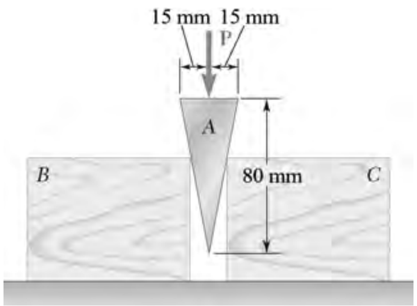
$$B_x + 1800\text{ N} - (1127.1\text{ N})\sin 21.31°$$

$$B_x = -1390.4\text{ N} \qquad \mathbf{B}_x = 1390\text{ N} \longleftarrow ◀$$

$$+\!\uparrow \Sigma F_y = 0: \quad B_y + R_C \cos 21.31° = 0$$

$$B_y + (1127.1\text{ N})\cos 21.31° = 0$$

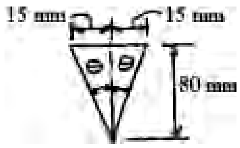
$$B_y = -1050\text{ N} \qquad \mathbf{B}_y = 1050\text{ N}\downarrow ◀$$



PROBLEM 8.140

A wedge A of negligible weight is to be driven between two 500 N blocks B and C resting on a horizontal surface. Knowing that the coefficient of static friction at all surfaces of contact is 0.35, determine the smallest force **P** required to start moving the wedge (*a*) if the blocks are equally free to move, (*b*) if block C is securely bolted to the horizontal surface.

SOLUTION

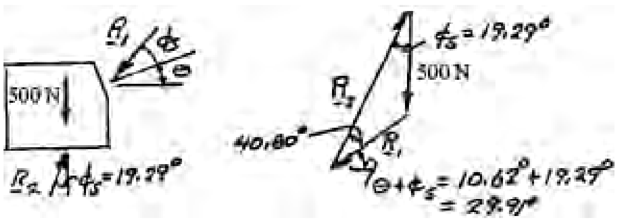


Wedge angle θ:

$$\theta = \tan^{-1}\frac{15 \text{ mm}}{80 \text{ mm}}$$

$$\theta = 10.62°$$

(*a*) Free body: Block B



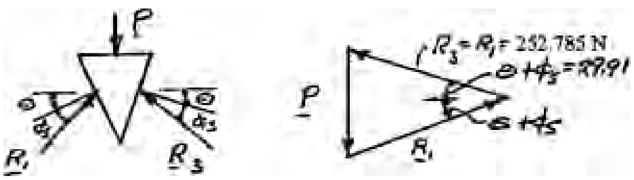
$$\phi_s = \tan^{-1} 0.35 = 19.29°$$

$$\frac{R_1}{\sin 19.29°} = \frac{500 \text{ N}}{\sin 40.80°}$$

$$R_1 = 252.785 \text{ N}$$

Free body: Wedge

By symmetry: $R_3 = R_1$



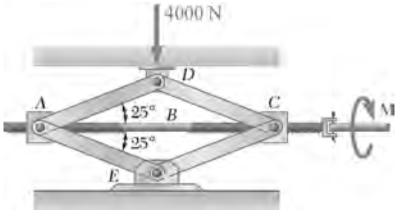
$$P = 2R_1 \sin(\theta + \phi_s) = 2(252.785)\sin 29.91°$$

$$P = 252.097 \text{ N}$$

$\mathbf{P} = 252 \text{ N} \downarrow$ ◀

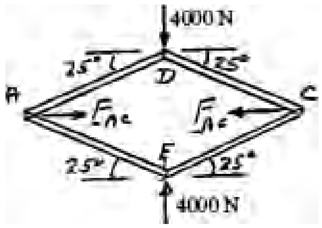
(*b*) Free bodies unchanged: Same result. $\mathbf{P} = 252 \text{ N} \downarrow$ ◀

PROBLEM 8.141



The position of the automobile jack shown is controlled by a screw *ABC* that is single-threaded at each end (right-handed thread at *A*, left-handed thread at *C*). Each thread has a pitch of 2 mm and a mean diameter of 7.5 mm. If the coefficient of static friction is 0.15, determine the magnitude of the couple **M** that must be applied to raise the automobile.

SOLUTION



Free body: Parts *A*, *D*, *C*, *E*

Two-force members

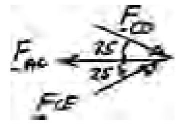
Joint *D*:

Symmetry: $F_{AD} = F_{CD}$

$$+\uparrow \Sigma F_y = 0: \quad 2F_{CD}\sin 25° - 4000 \text{ N} = 0$$

$$F_{CD} = 4732.4 \text{ N}$$

Joint *C*:



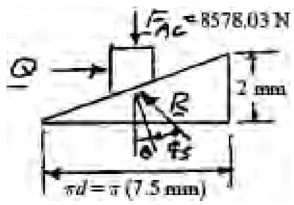
Symmetry: $F_{CE} = F_{CD}$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 2F_{CD}\cos 25° - F_{AC} = 0$$

$$F_{AC} = 2(4732.4 \text{ N})\cos 25°$$

$$F_{AC} = 8578.03 \text{ N}$$

Block-and-incline analysis of *one* screw:

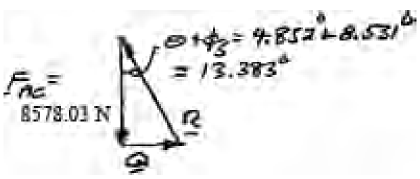


$$\tan\theta = \frac{2 \text{ mm}}{\pi(7.5 \text{ mm})}$$

$$\theta = 4.852°$$

$$\phi_s = \tan^{-1} 0.15$$

$$= 8.531°$$



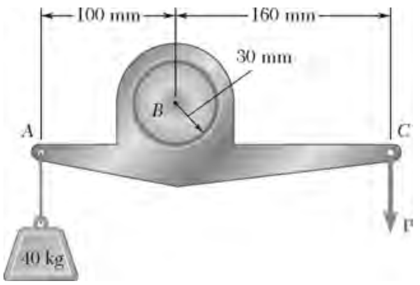
$$Q = (8578.03 \text{ N})\tan 13.383°$$

$$Q = 2040.89 \text{ N}$$

But, we have *two* screws: $\text{Torque} = 2Qr = 2(2040.89 \text{ N})\left(\frac{7.5 \text{ mm}}{2}\right)$

$$= 15306.7 \text{ N}\cdot\text{mm} = 15.3067 \text{ N}\cdot\text{m}$$

$$\text{Torque} = 15.31 \text{ N}\cdot\text{m} \blacktriangleleft$$

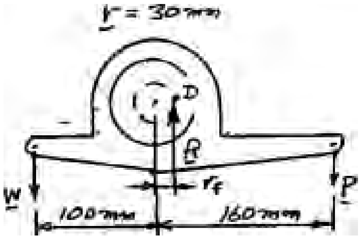


PROBLEM 8.142

A lever of negligible weight is loosely fitted onto a 30-mm-radius fixed shaft as shown. Knowing that a force **P** of magnitude 275 N will just start the lever rotating clockwise, determine (*a*) the coefficient of static friction between the shaft and the lever, (*b*) the smallest force **P** for which the lever does not start rotating counterclockwise.

SOLUTION

(*a*) Impending motion ↻



$$W = (40 \text{ kg})(9.81 \text{ m/s}^2) = 392.4 \text{ N}$$

$$+\circlearrowright \Sigma M_D = 0: \quad P(160 - r_f) - W(100 + r_f) = 0$$

$$r_f = \frac{160P - 100W}{P + W}$$

$$r_f = \frac{(160 \text{ mm})(275 \text{ N}) - (100 \text{ mm})(392.4 \text{ N})}{275 \text{ N} + 392.4 \text{ N}}$$

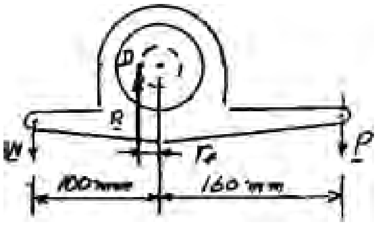
$$r_f = 7.132 \text{ mm}$$

$$r_f = r \sin\phi_s = r\mu_s$$

$$\mu_s = \frac{r_f}{r} = \frac{7.132 \text{ mm}}{30 \text{ mm}} = 0.2377$$

$\mu_s = 0.238$ ◀

(*b*) Impending motion ↺



$$r_f = r \sin\phi_s = r\mu_s$$

$$= (30 \text{ mm})(0.2377)$$

$$r_f = 7.132 \text{ mm}$$

$$+\circlearrowright \Sigma M_D = 0: \quad P(160 + r_f) - W(100 - r_f) = 0$$

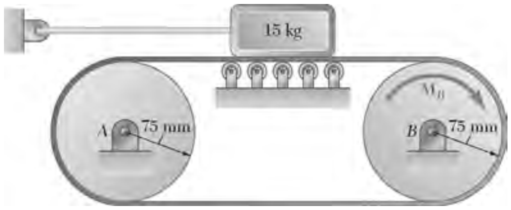
$$P = W\frac{100 - r_f}{160 + r_f}$$

$$P = (392.4 \text{ N})\frac{100 \text{ mm} - 7.132 \text{ mm}}{160 \text{ mm} + 7.132 \text{ mm}}$$

$$P = 218.04 \text{ N}$$

P = 218 N ↓ ◀

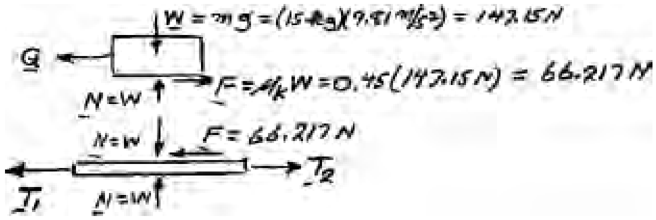
PROBLEM 8.143



A couple $\mathbf{M}_B$ is applied to the drive drum B to maintain a constant speed in the polishing belt shown. Knowing that $\mu_k = 0.45$ between the belt and the 15-kg block being polished and $\mu_s = 0.30$ between the belt and the drive drum B, determine (*a*) the couple $\mathbf{M}_B$, (*b*) the minimum tension in the lower portion of the belt if no slipping is to occur between the belt and the drive drum.

SOLUTION

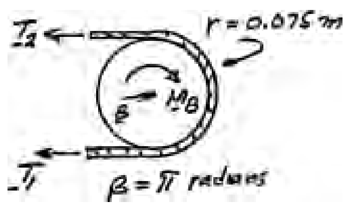
<u>Block</u>:



<u>Portion of belt located under block</u>:

$$\xrightarrow{+} \Sigma F_x = 0: \quad T_2 - T_1 - 66.217 \text{ N} = 0 \qquad (1)$$

<u>Drum *B*</u>:



$$\frac{T_2}{T_1} = e^{\mu_s \pi} = e^{0.3\pi} = 2.5663$$

$$T_2 = 2.5663 T_1 \qquad (2)$$

Eq. (1): $$2.5663 T_1 - T_1 - 66.217 \text{ N} = 0$$

$$1.5663 T_1 = 66.217 \text{ N}$$

$$T_1 = 42.276 \text{ N} \qquad T_{\min} = 42.3 \text{ N} \blacktriangleleft$$

Eq. (2): $$T_2 = 2.5663(42.276 \text{ N}) = 108.493 \text{ N}$$

$$\overset{+}{\circlearrowright} \Sigma M_B = 0: \quad M_B - (108.493 \text{ N})(0.075 \text{ m}) + (42.276 \text{ N})(0.075 \text{ m}) = 0$$

$$M_B = 4.966 \text{ N} \cdot \text{m} \qquad \mathbf{M}_B = 4.97 \text{ N} \cdot \text{m} \circlearrowright \blacktriangleleft$$